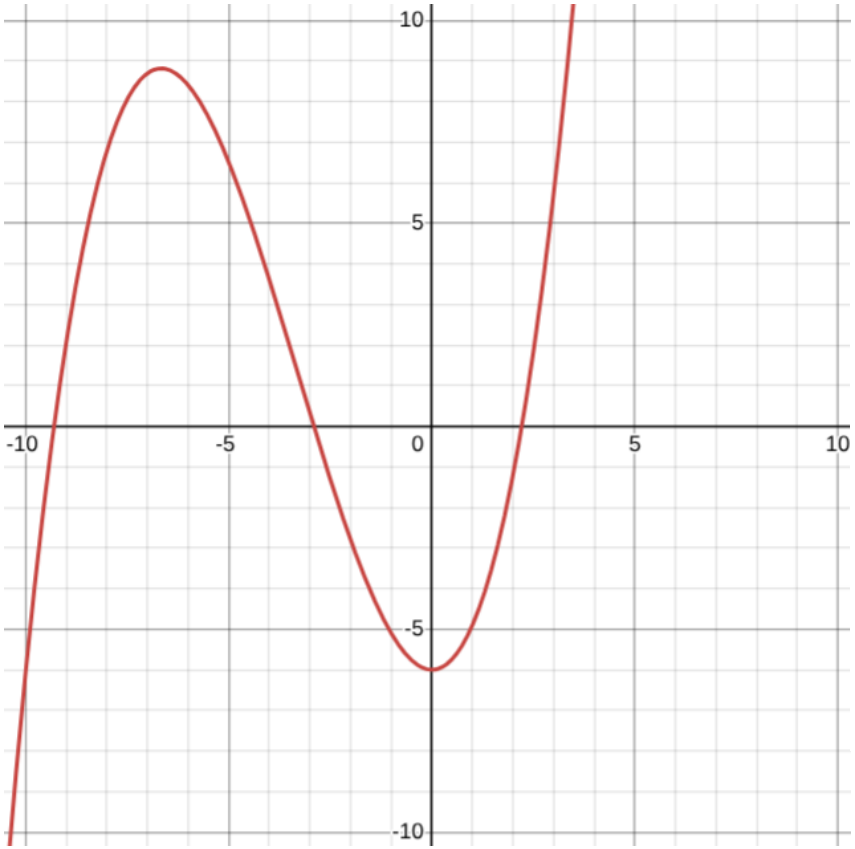
Graph Theory: The Study of Relationships

Girls Talk Math

Brandon Kolstoe

University of Maryland - College Park

July 9, 2025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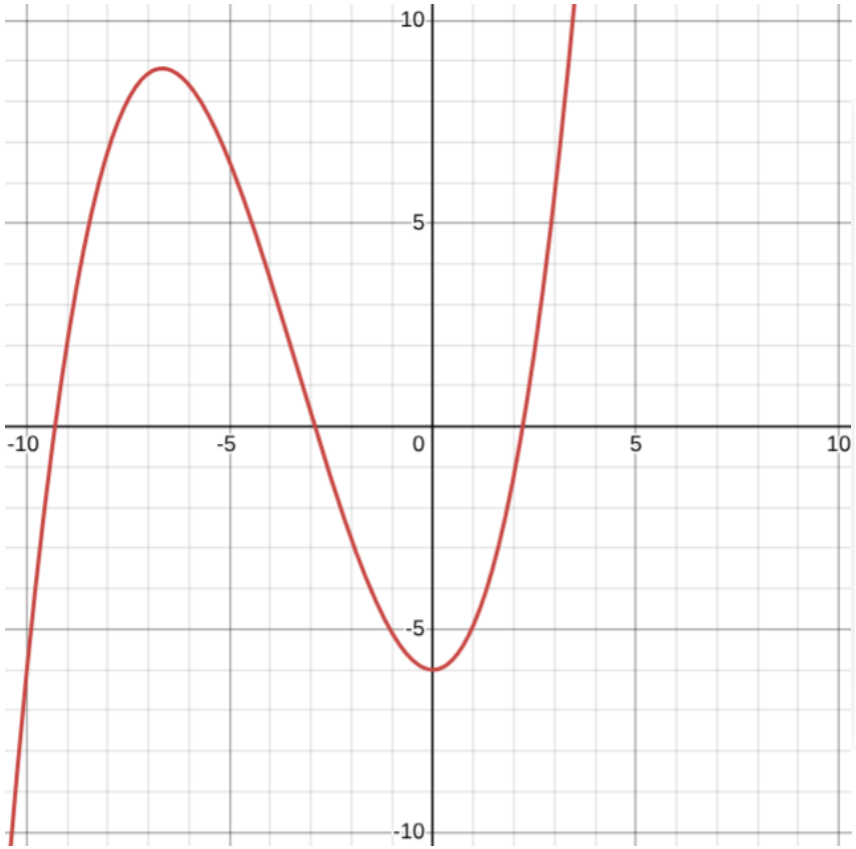
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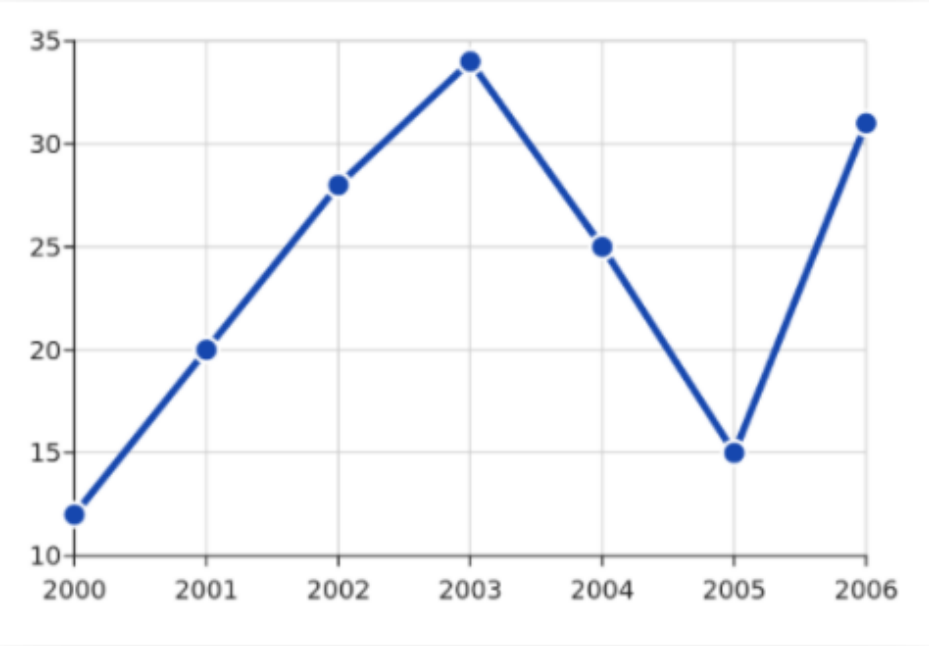
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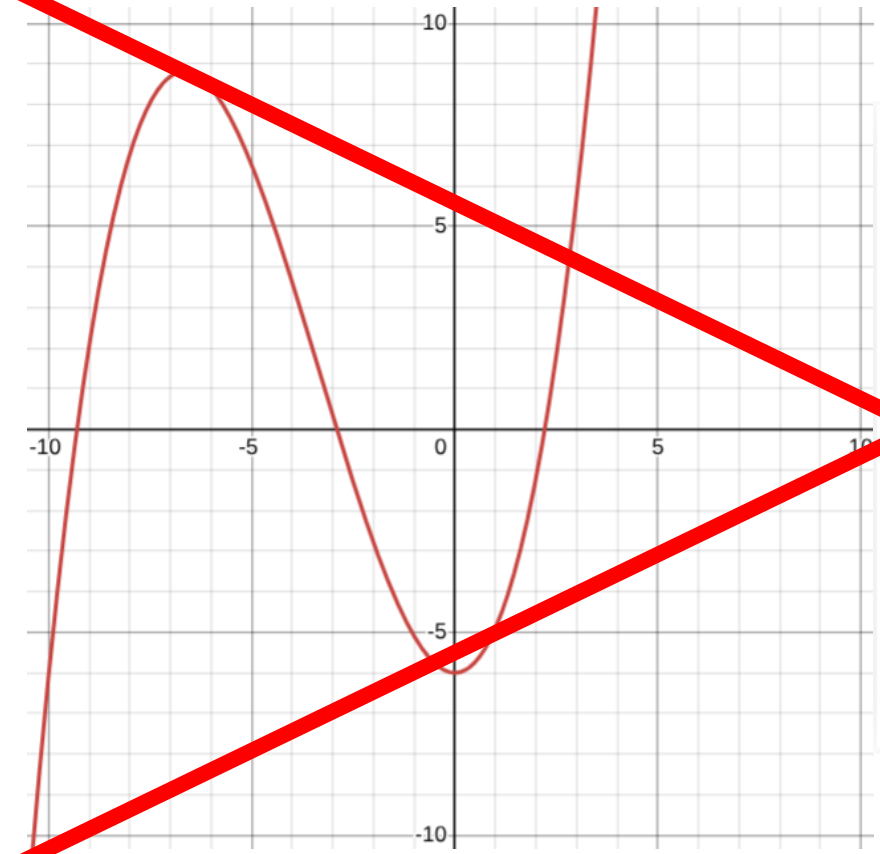
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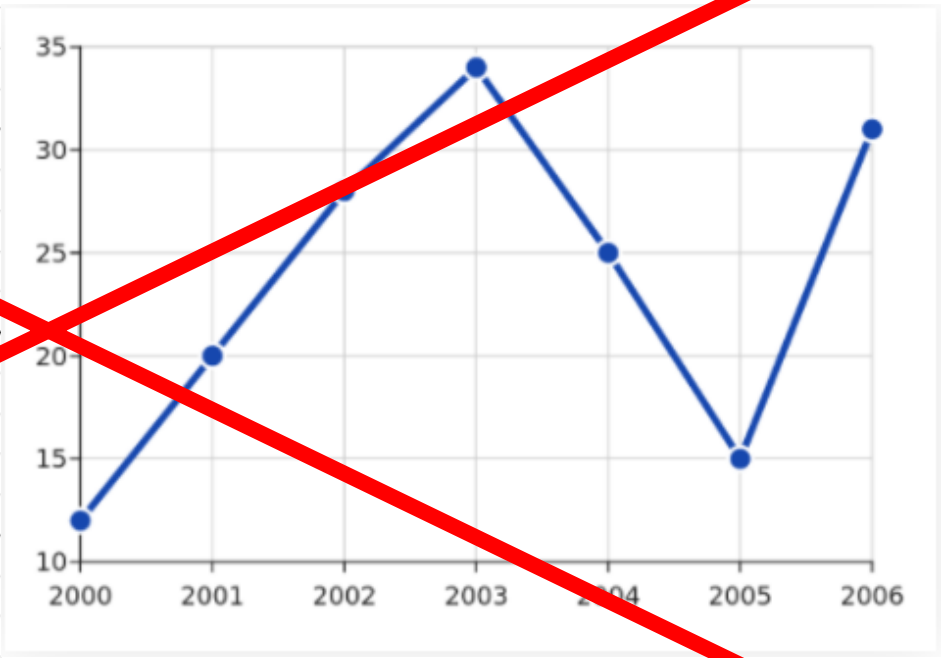
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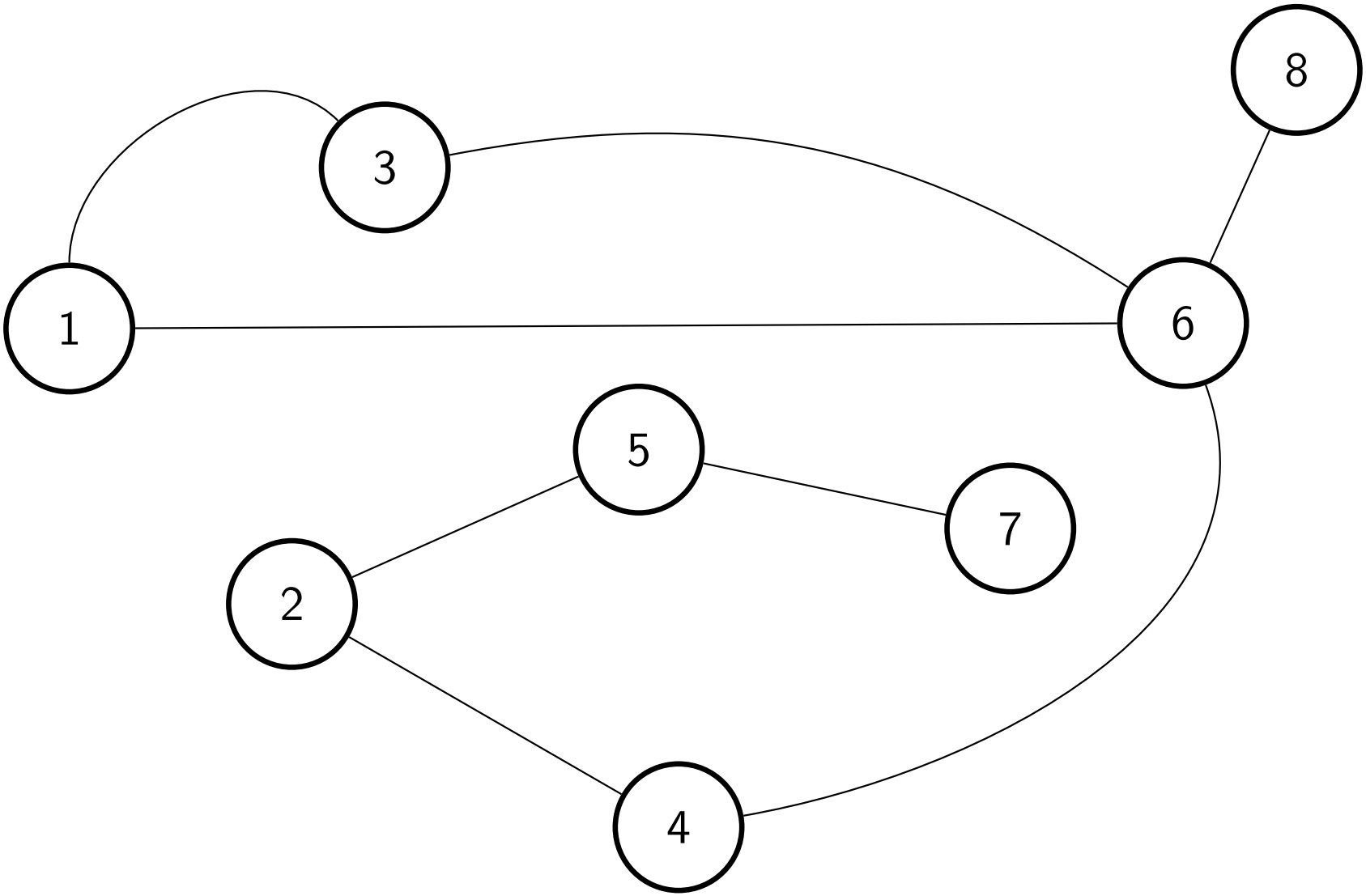
Graphs



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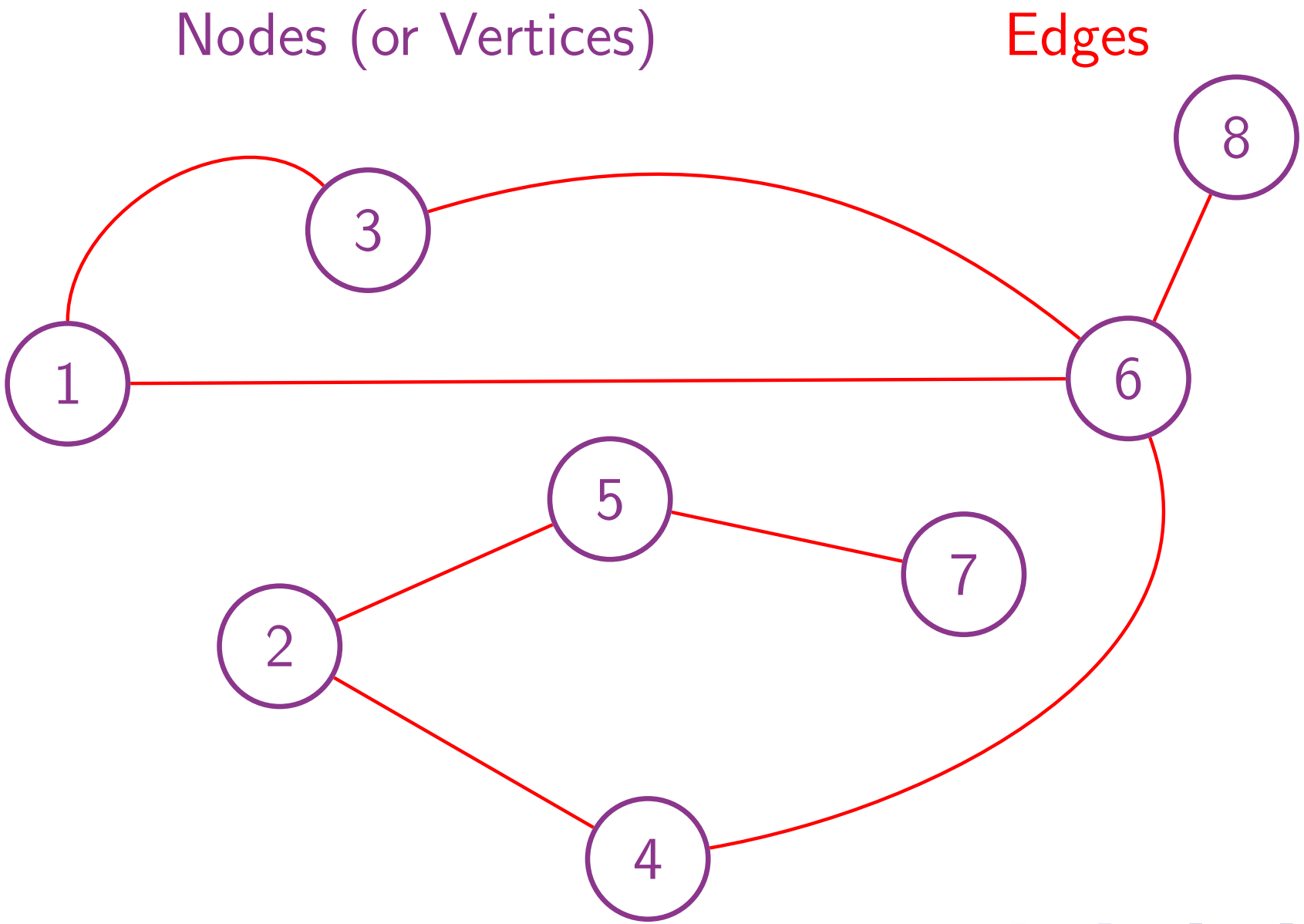
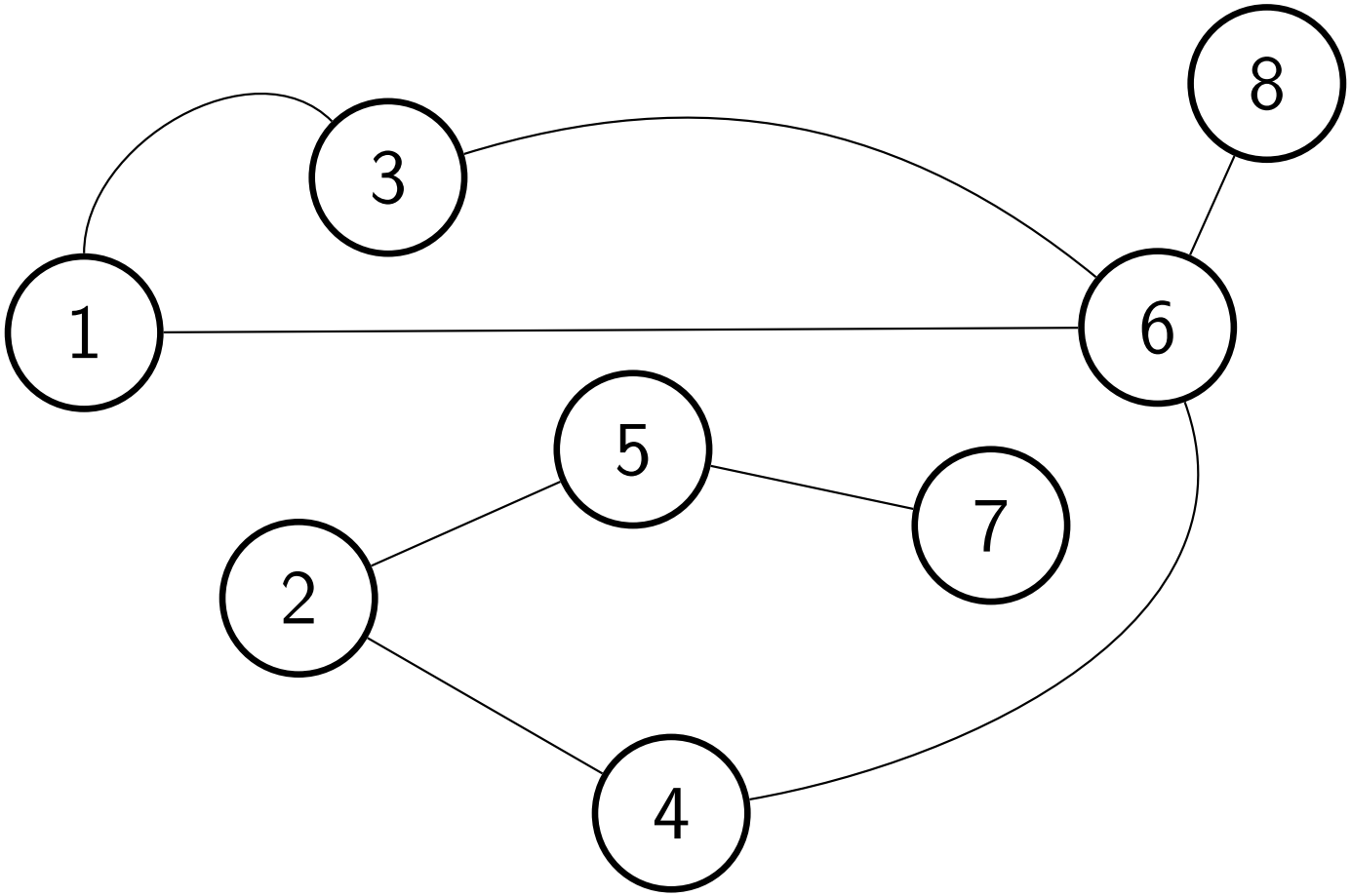


Graphs

- Nodes represent some sort of object
- Edges represent some relationship between 2 nodes.



Drawing a Graph

Let's represent the following as a graph, where:

- Nodes are people
- Edges exist between two people if they know each other.

Six people are at a party: Alice, Bob, Carol, David, Erin, and Frank.

- Alice knows Bob and David.
- Carol knows Bob, David and Frank.
- David knows Alice, Carol, and Frank

Drawing a Graph

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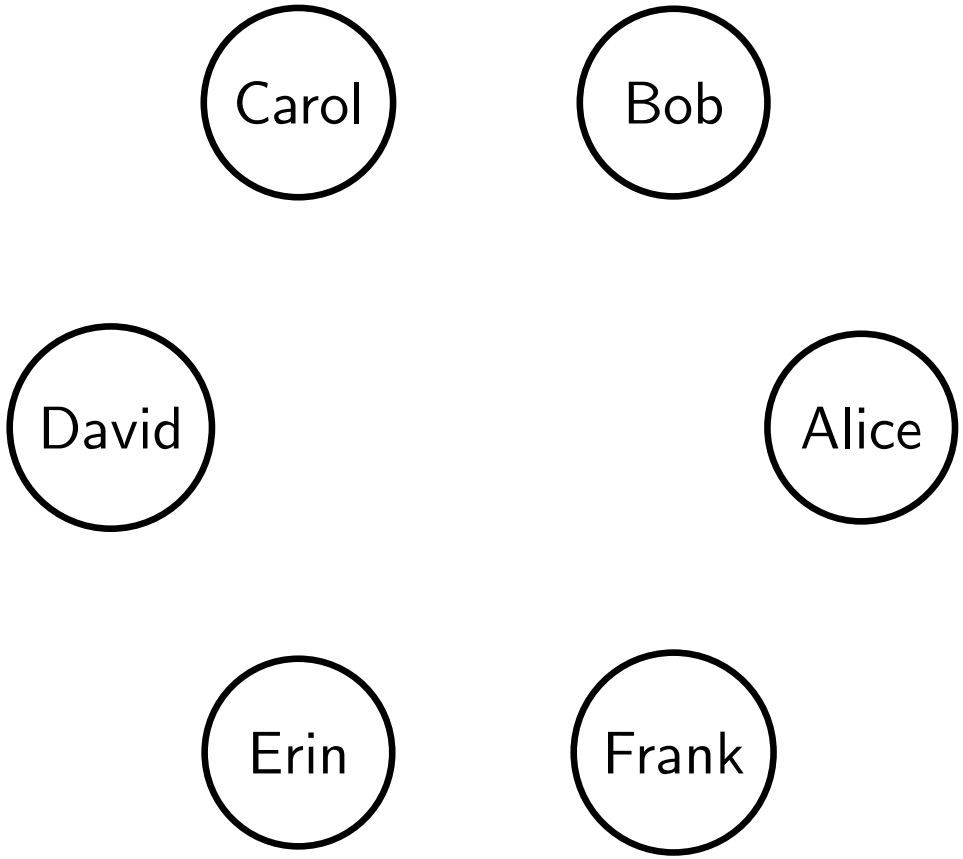

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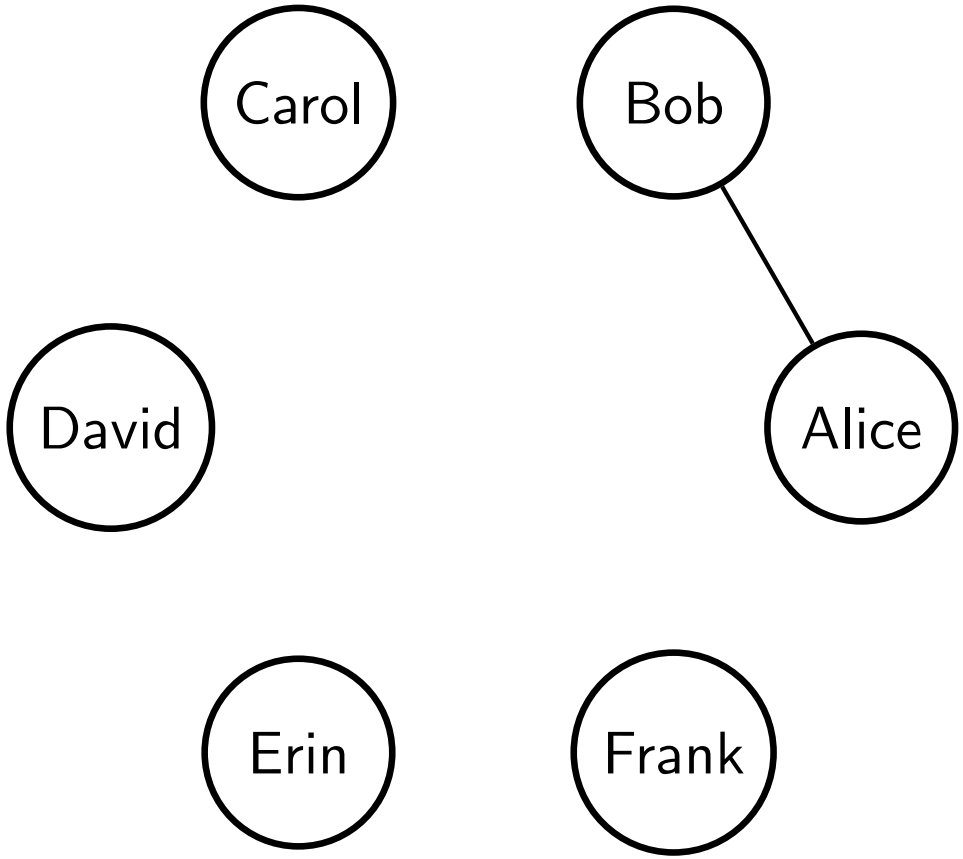

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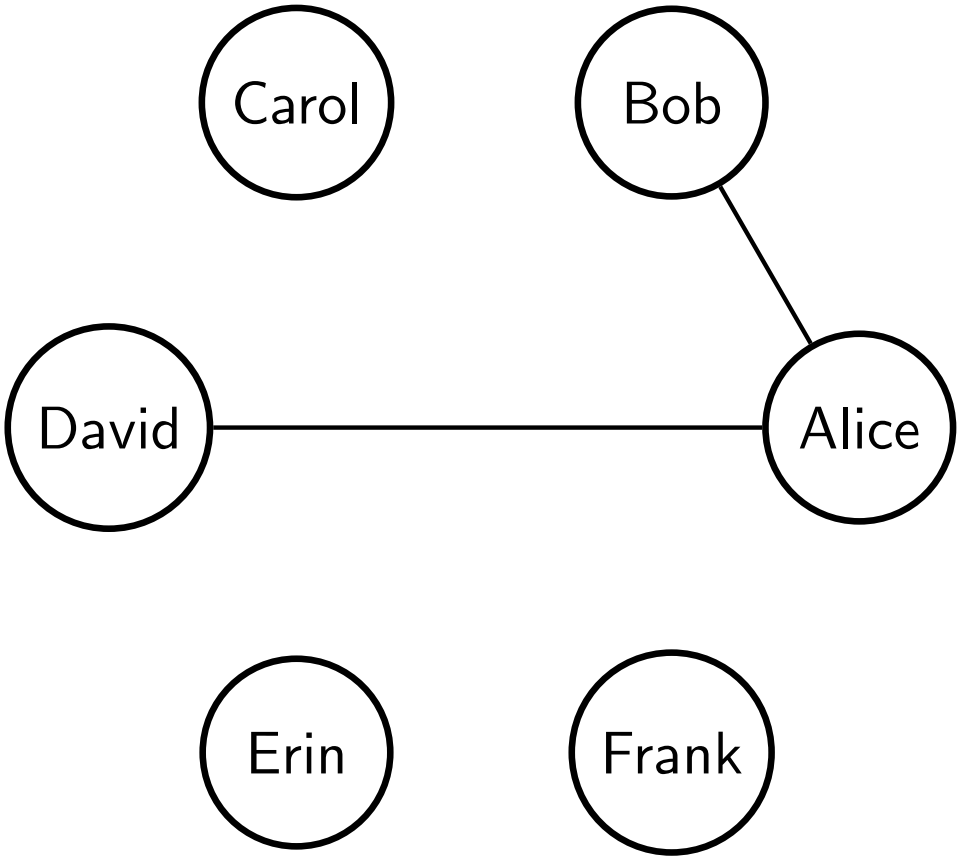

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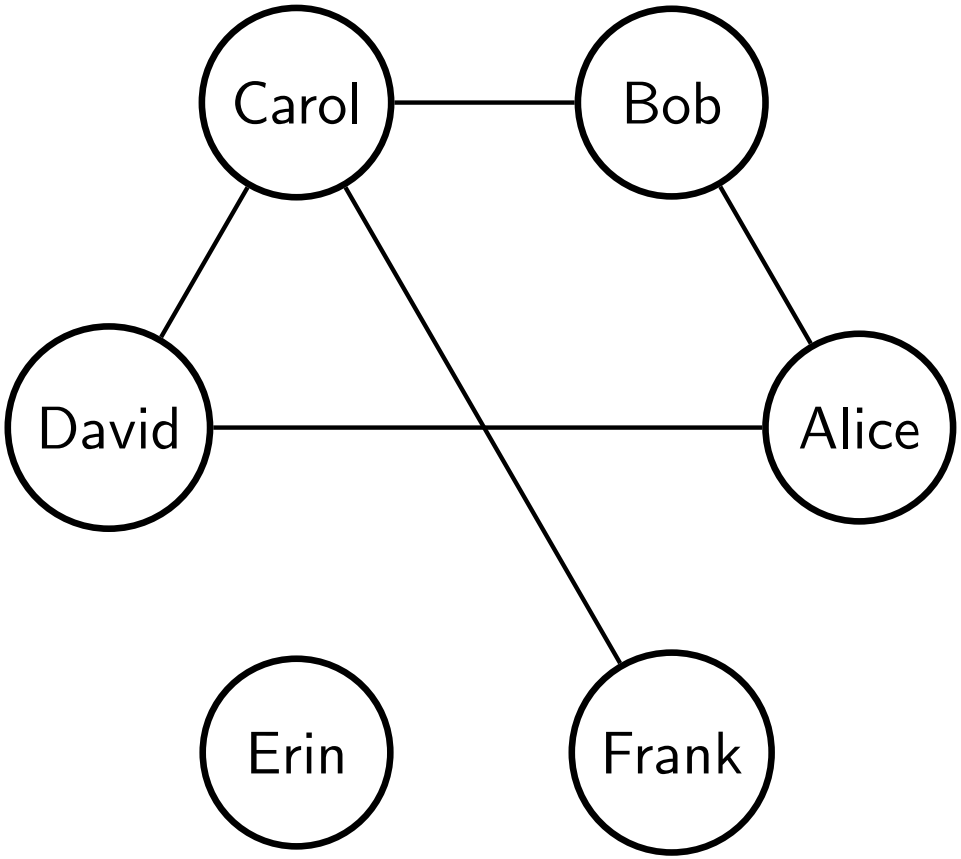

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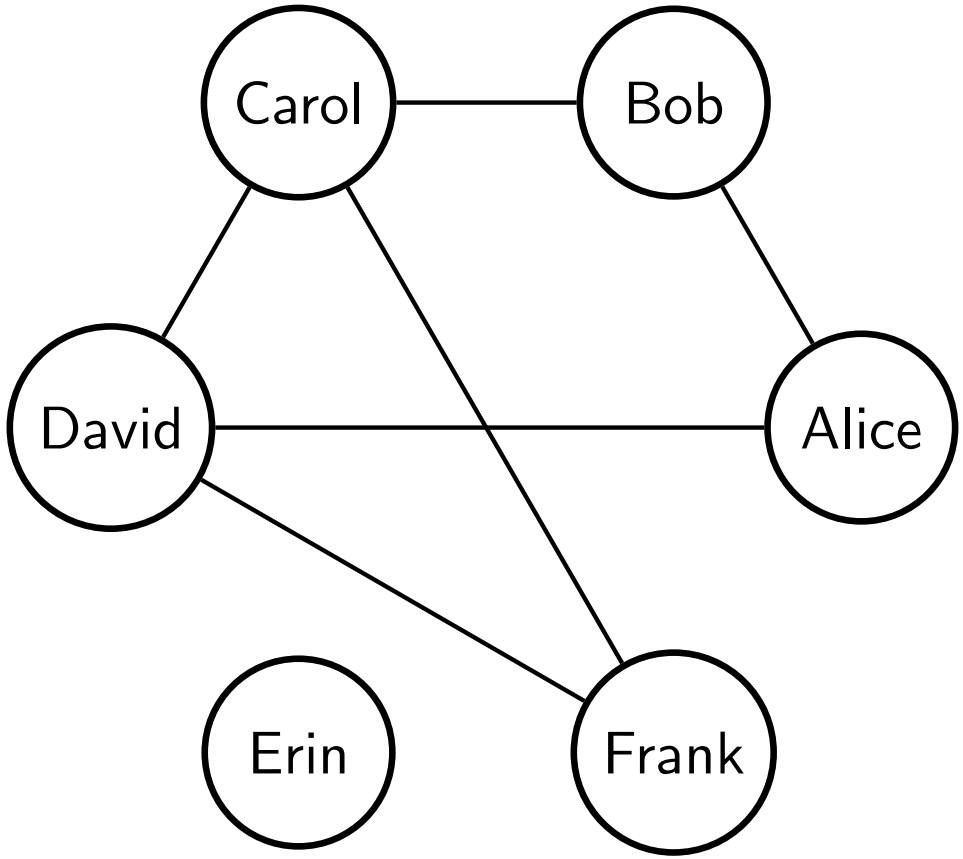
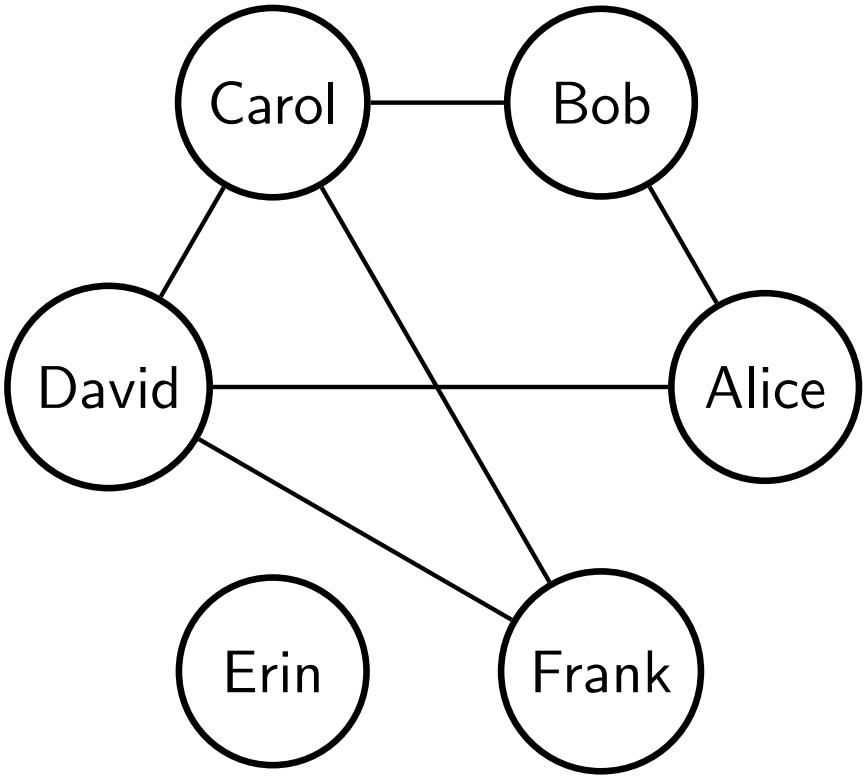
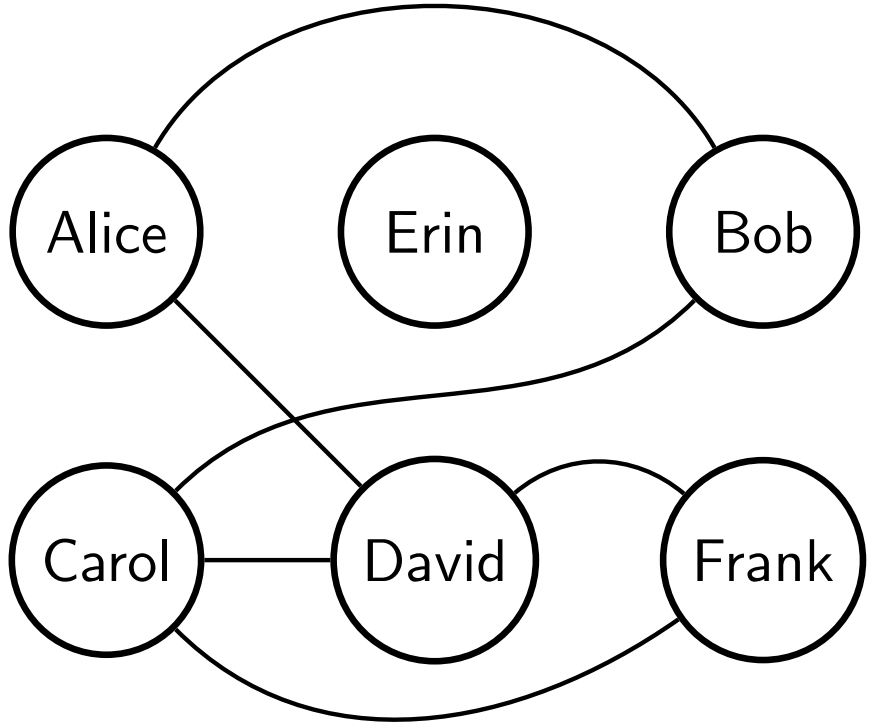


Drawing a Graph

What matters in drawing a graph is what the nodes are and whether an edge exists; it doesn't matter how the graph is drawn.





These are the same graph!

Graphs in the Real World

You can represent a road system as a graph.

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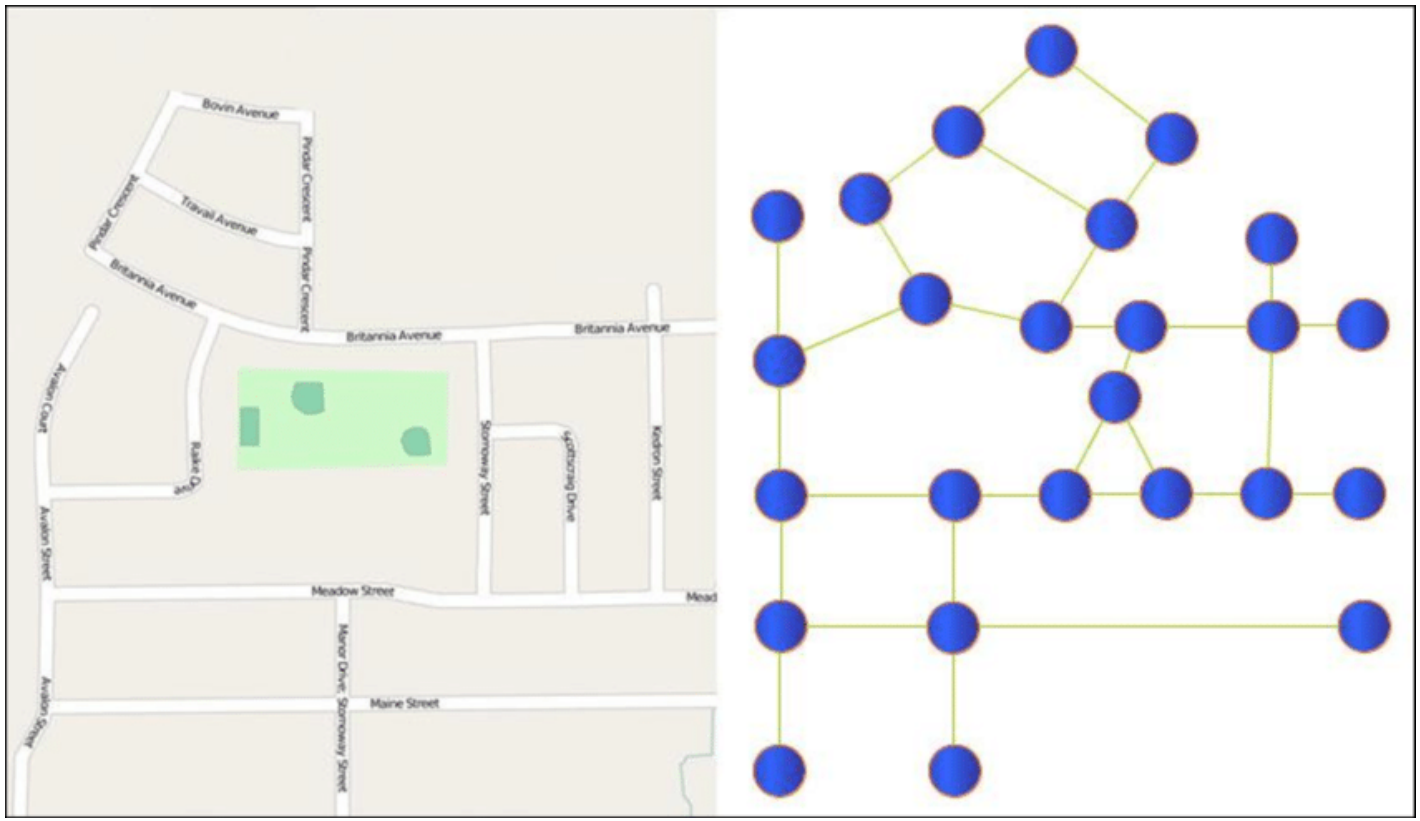

A. Alganas. "Social-Based Trustworthy Data Forwarding in Vehicular Delay Tolerant Networks". MA thesis. University of Ontario Institute of Technology, 2011.

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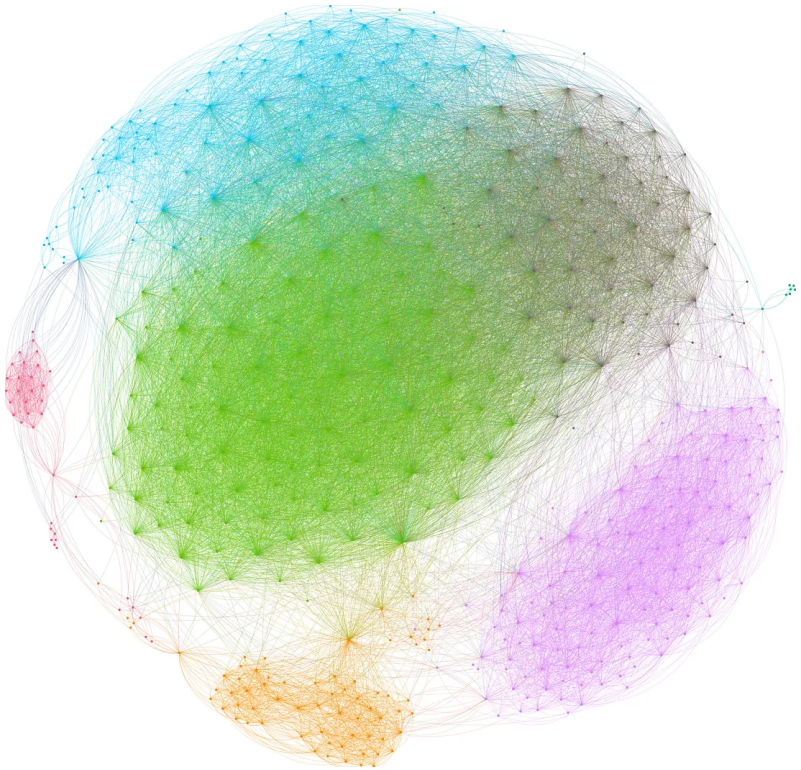

Giraffael. *Social Network Analysis on My Instagram Connections Graph*. 2024.

Graphs in the Real World: Gerrymandering

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Every 10 years, states need to draw new maps for their Congressional districts (where each Representative is elected).

- They are given a certain number of seats in the House of Representatives according to each state's population.
- Each state government needs to divide their state into that number of districts, all with approximately equal population.

From `https://en.wikipedia.org/wiki/Maryland%27s_congressional_districts`

Graphs in the Real World: Gerrymandering

The new maps drawn may be a **gerrymander**

- A gerrymander is when a district map leads to an unfair advantage for one group (say a political party, or incumbents).

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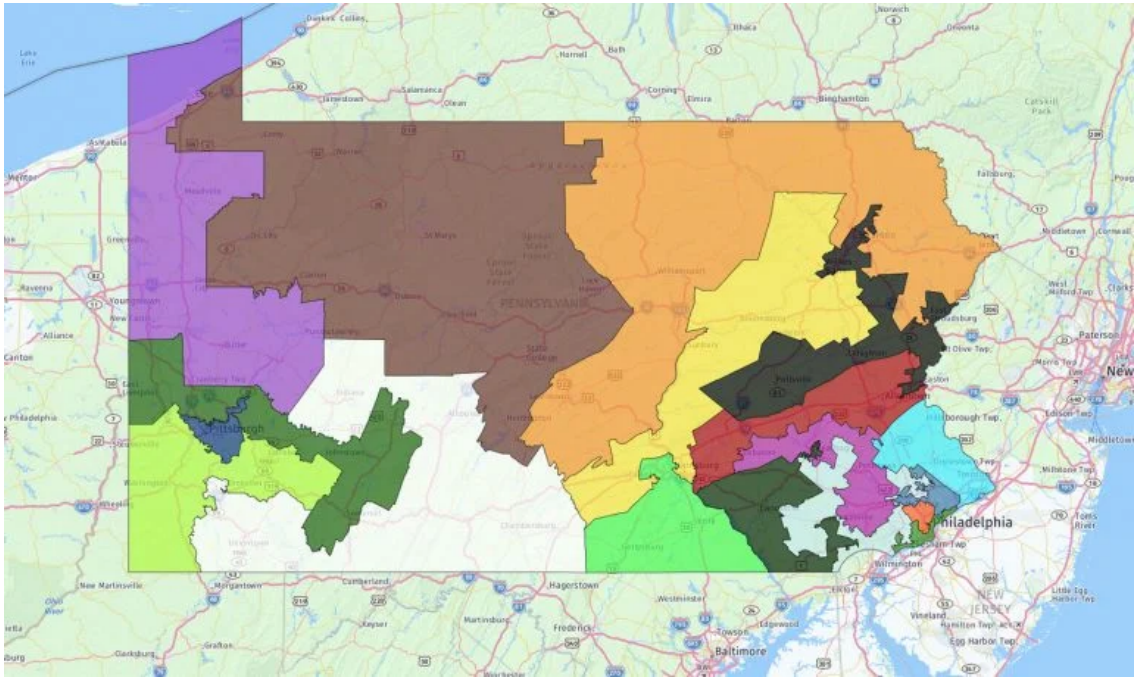

Figure 1: Pennsylvania district map enacted in 2011. This map seemed to unfairly help Republicans (who won 13 of the 18 districts).

E. Previti. "Day 2 testimony in gerrymandering lawsuit delves deeper into making Pa.'s congressional map" . In: *WHYY* (2017).

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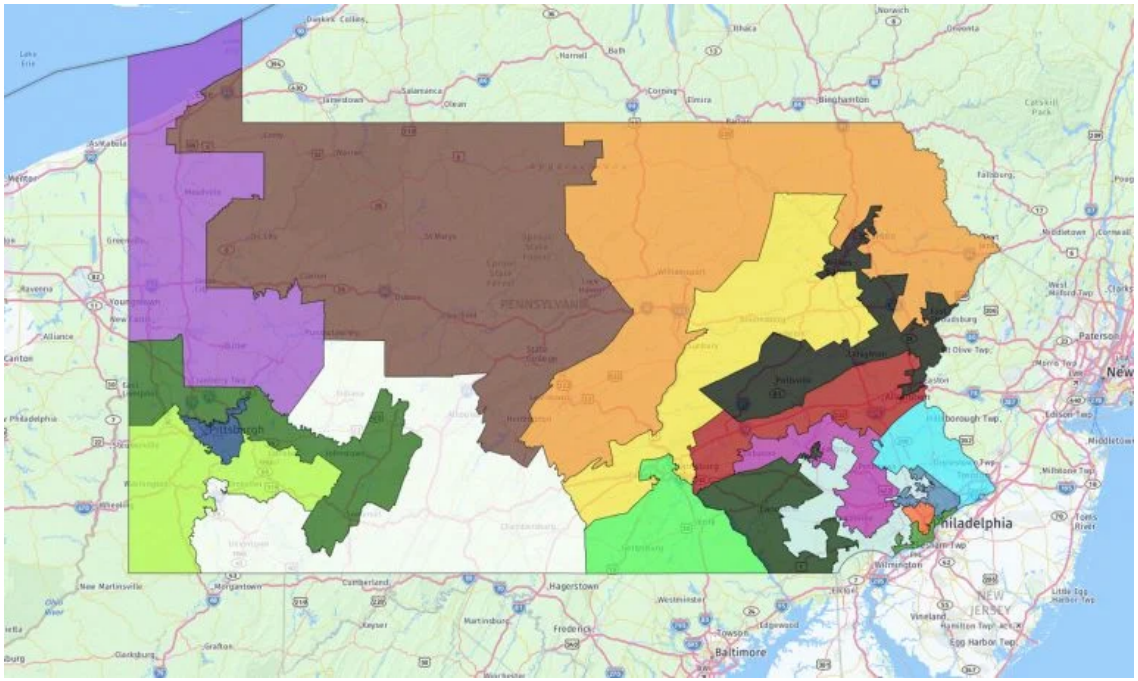

Figure 1: Pennsylvania district map enacted in 2011. This map seemed to unfairly help Republicans (who won 13 of the 18 districts).

Is there a good way to say that a district map is a gerrymander?

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Graphs in the Real World: Gerrymandering

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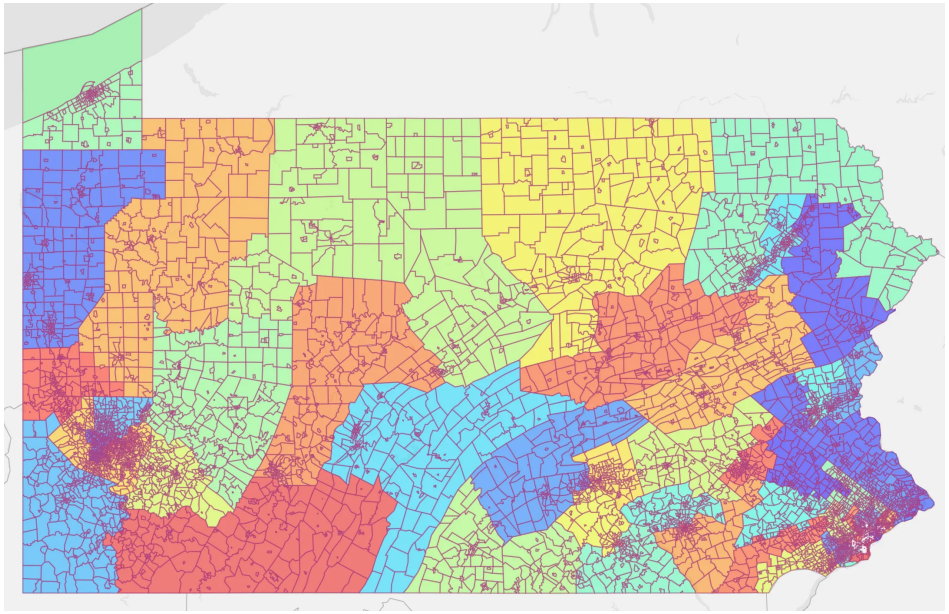
Districts are created by combining neighboring **precincts**, which are geographic areas in which every person living there votes in the same place.



From `https://davesredistricting.org/maps#home`.

Graphs in the Real World: Gerrymandering

Moon Duchin

Districts are created by combining neighboring **precincts**, which are geographic areas in which every person living there votes in the same place.

We can make a graph of a state where nodes are the precincts and edges exist if the precincts border each other.

M. Aminian. "The Impact of Generative Models and Social Choice Theory in Elections". In: *SIAM News* 58.2 (2025).

Graphs in the Real World: Gerrymandering

A district map is just a division of this graph into a bunch of smaller graphs, each with approximately the same population.

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A computer program (which uses some graph theory techniques) can use the graph to create tens of thousands of possible maps.
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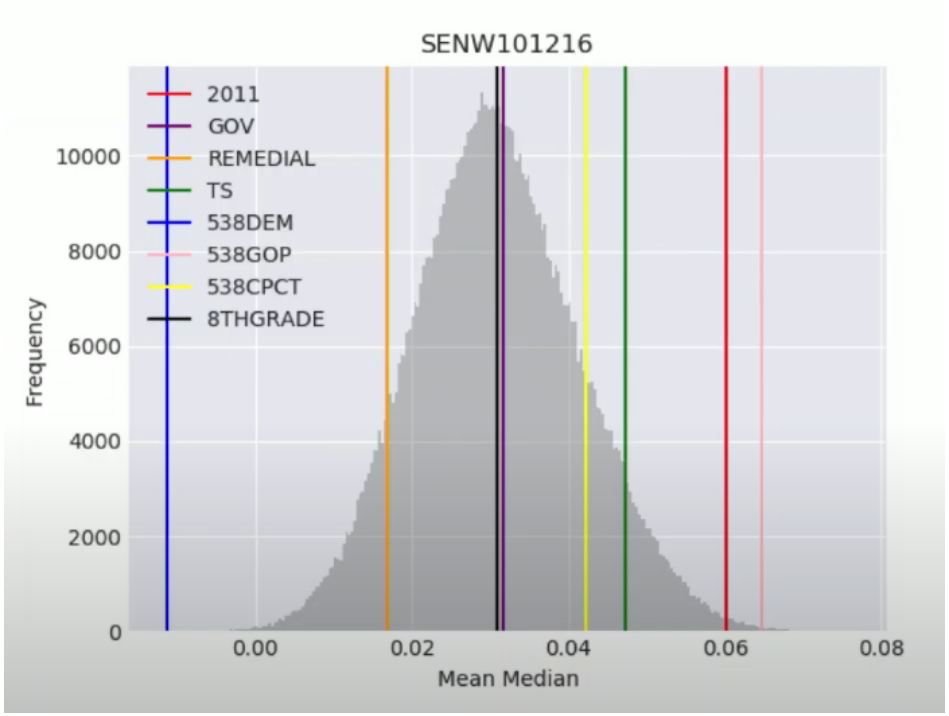
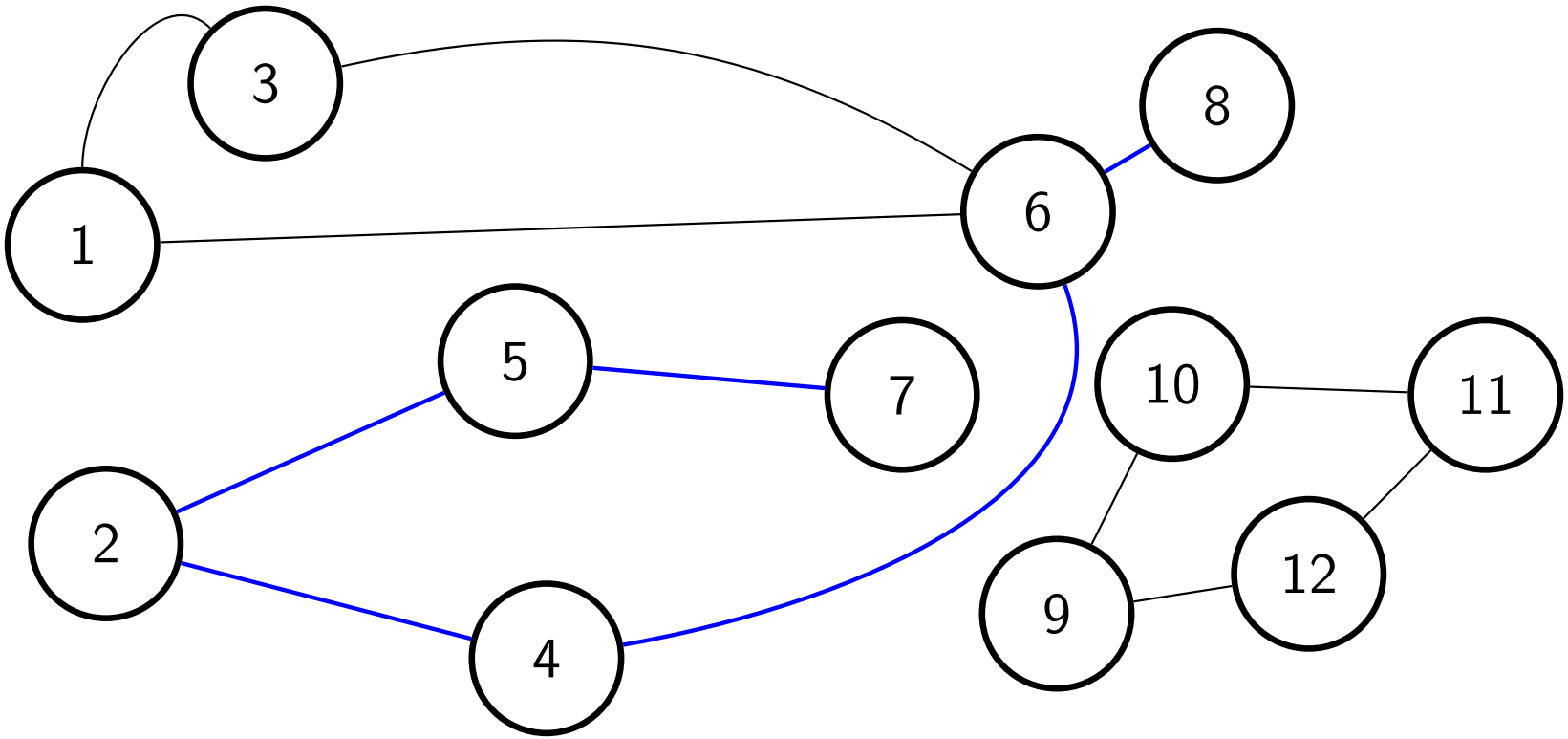


(from `https://www.youtube.com/watch?v=ykk4ddjukoA`)

Parts of Graphs: Paths

Path: Can you go from one node to another along edges?

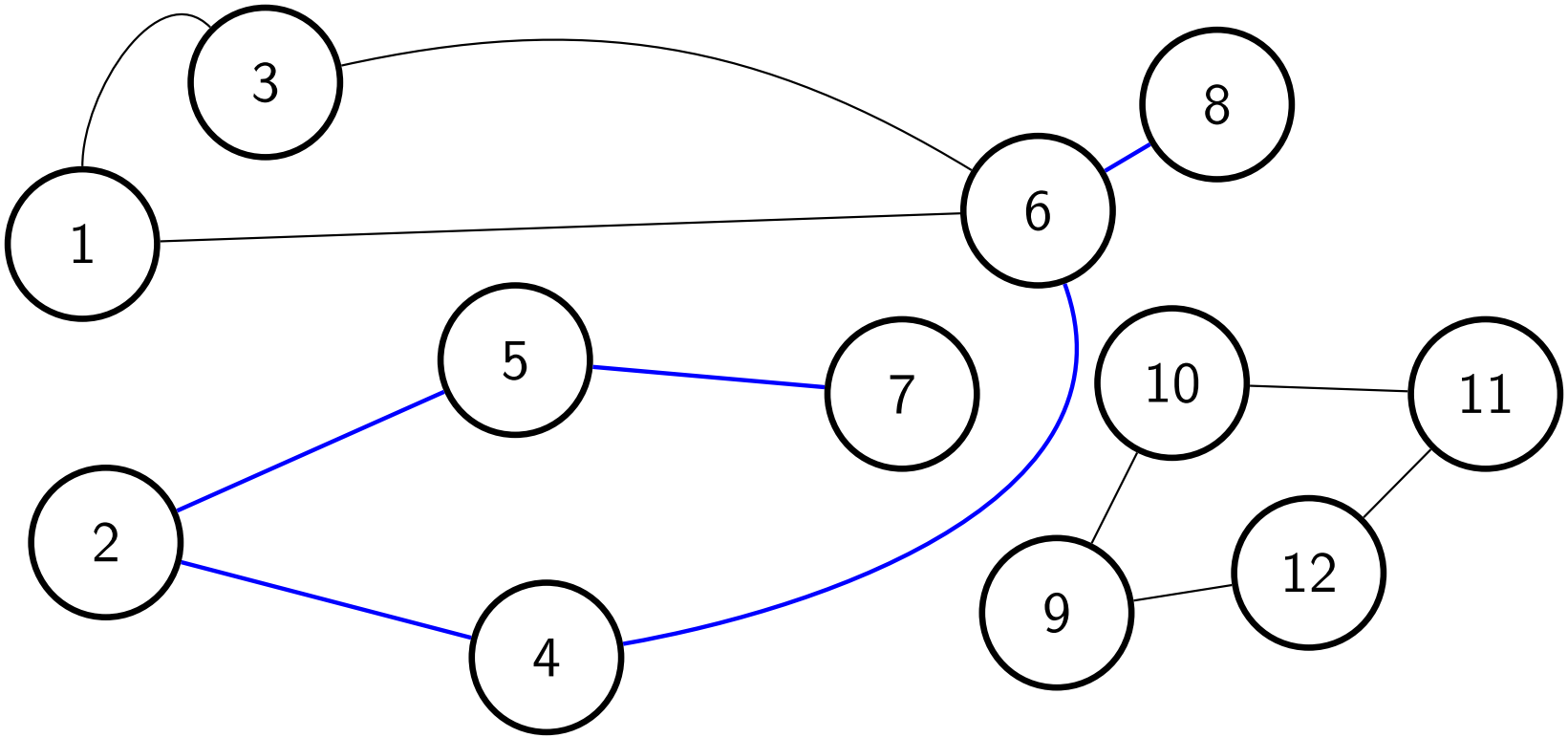
- There is a path from node 7 to node 8.



Parts of Graphs: Paths

Path: Can you go from one node to another along edges?

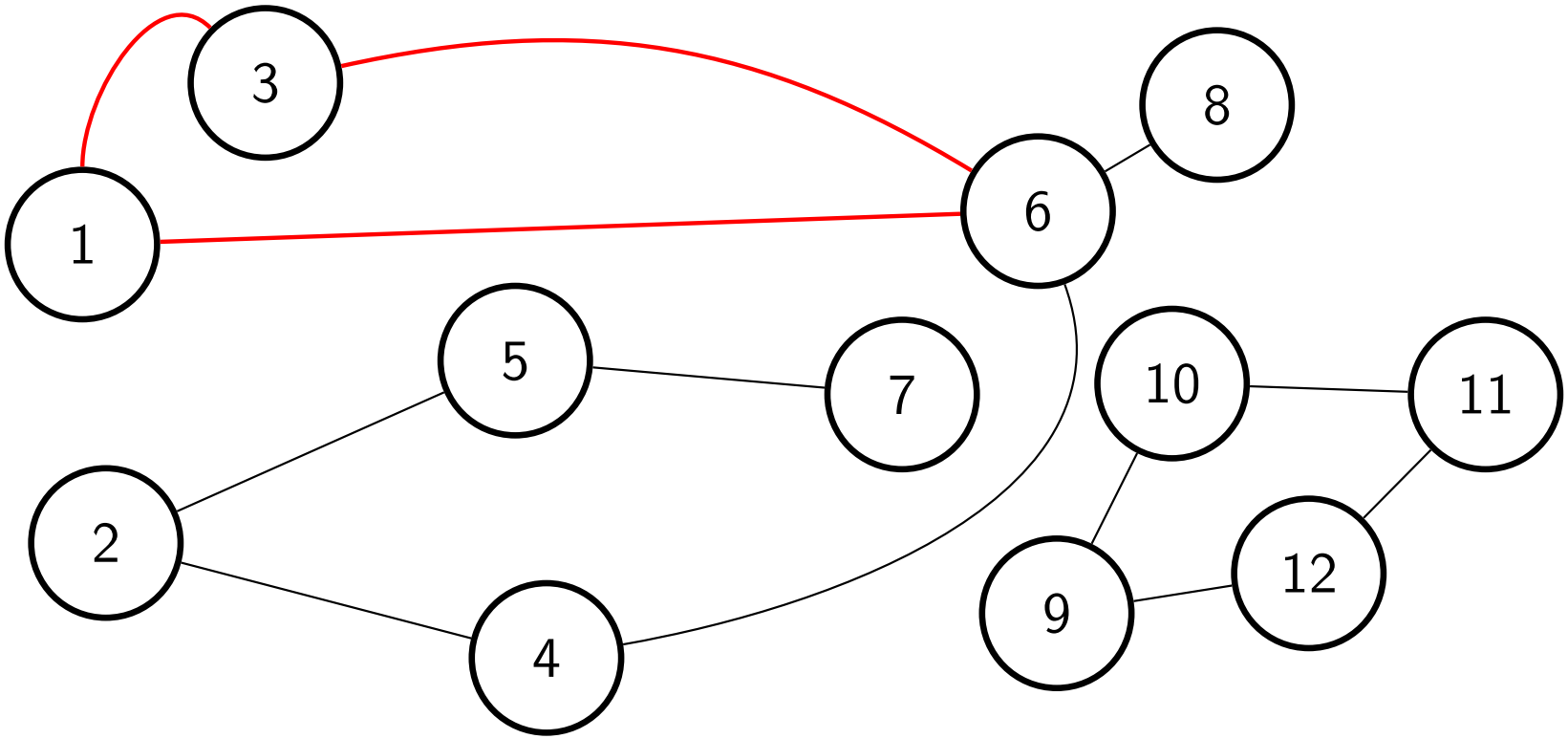
- There is a path from node 7 to node 8.
- There is no path from node 7 to node 9.



Parts of Graphs: Loops

Loop (or cycle): Can you reach your starting node without using the same path twice?

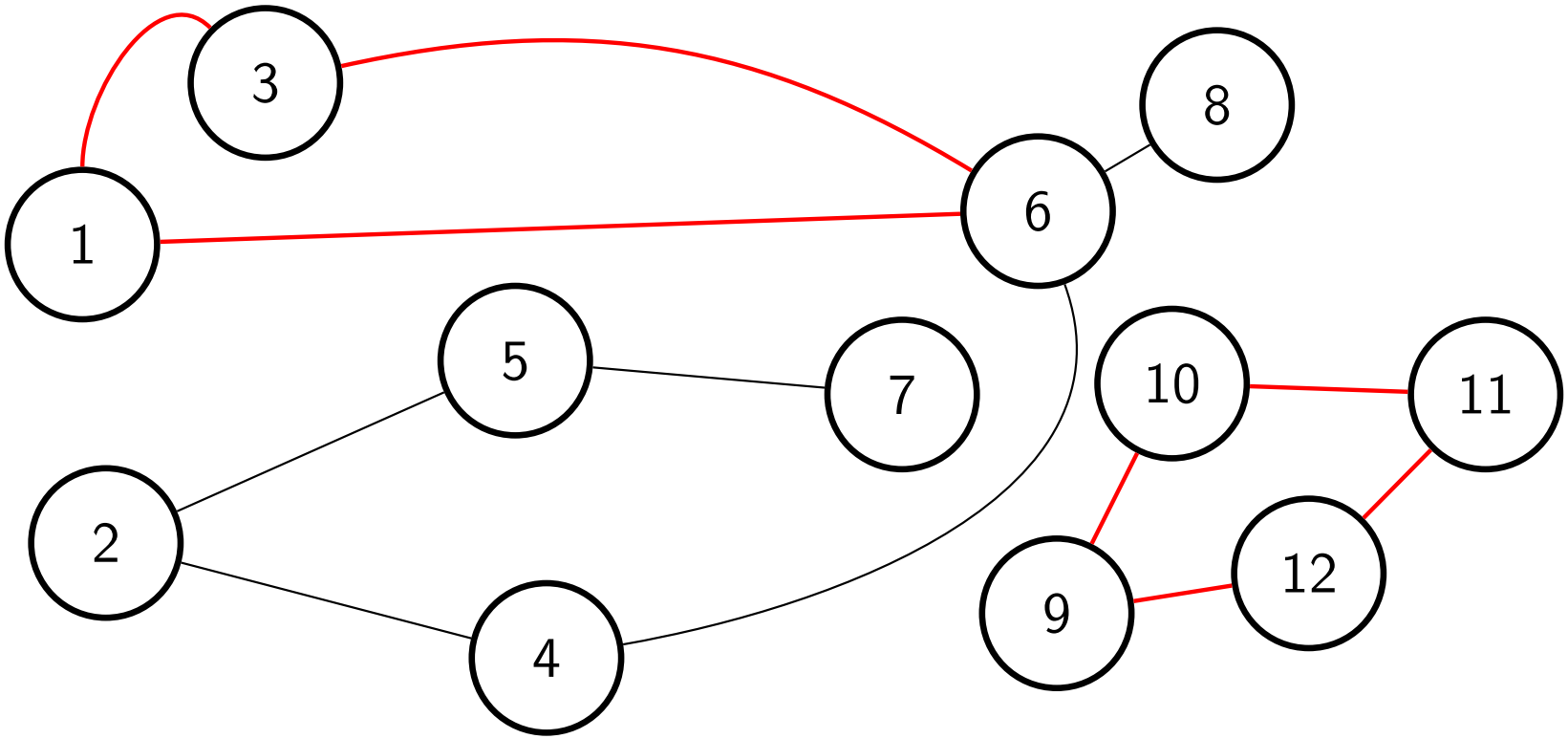
- There is a loop connecting nodes 1, 3, and 6.



Parts of Graphs: Loops

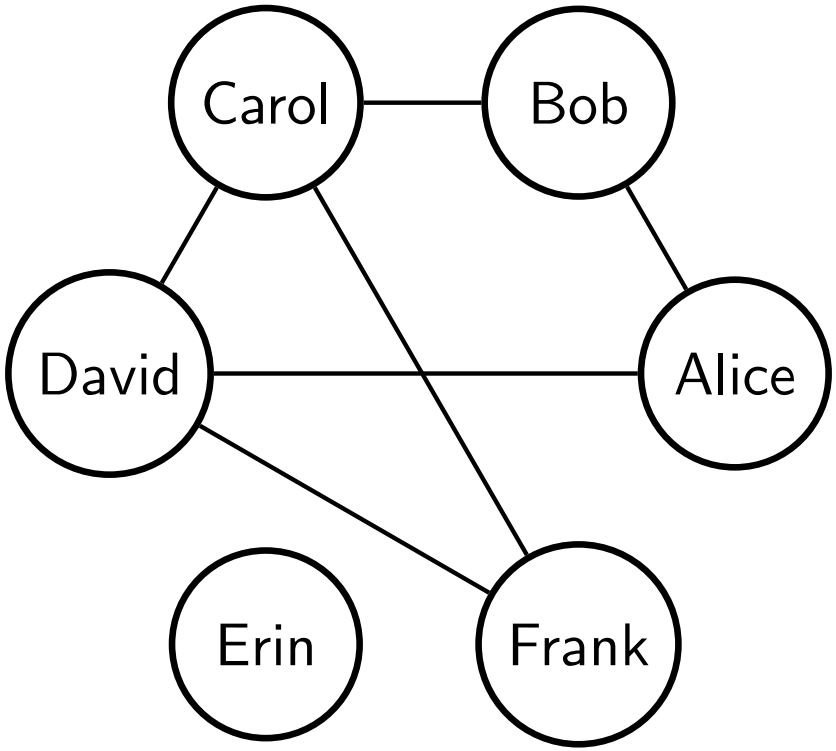
Loop (or cycle): Can you reach your starting node without using the same path twice?

- There is a loop connecting nodes 1, 3, and 6.
- There is a loop connecting nodes 9 through 12.



Parts of Graphs: Paths and Loops

Is there a path from Frank to Alice?



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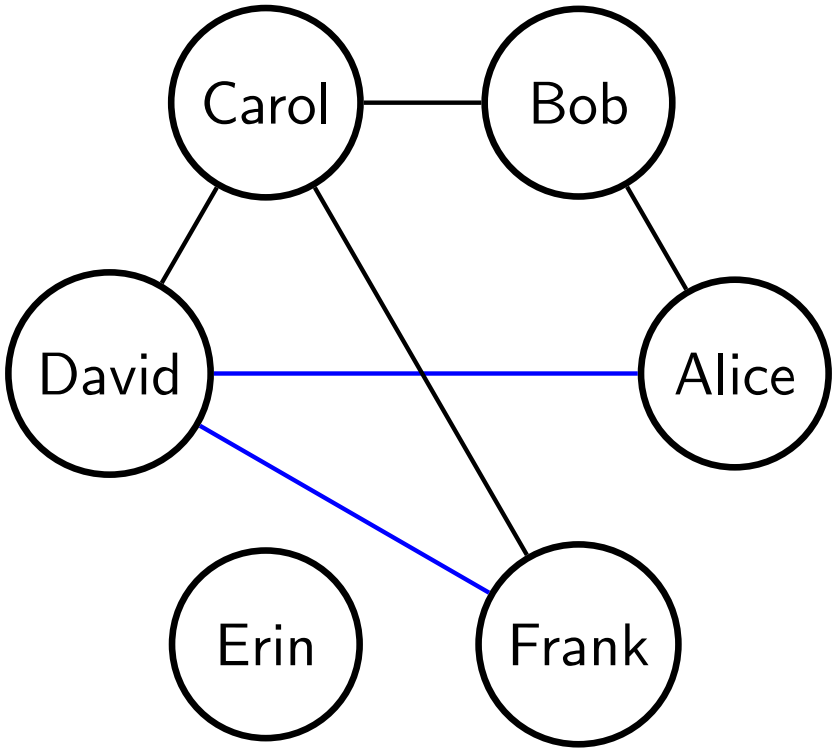

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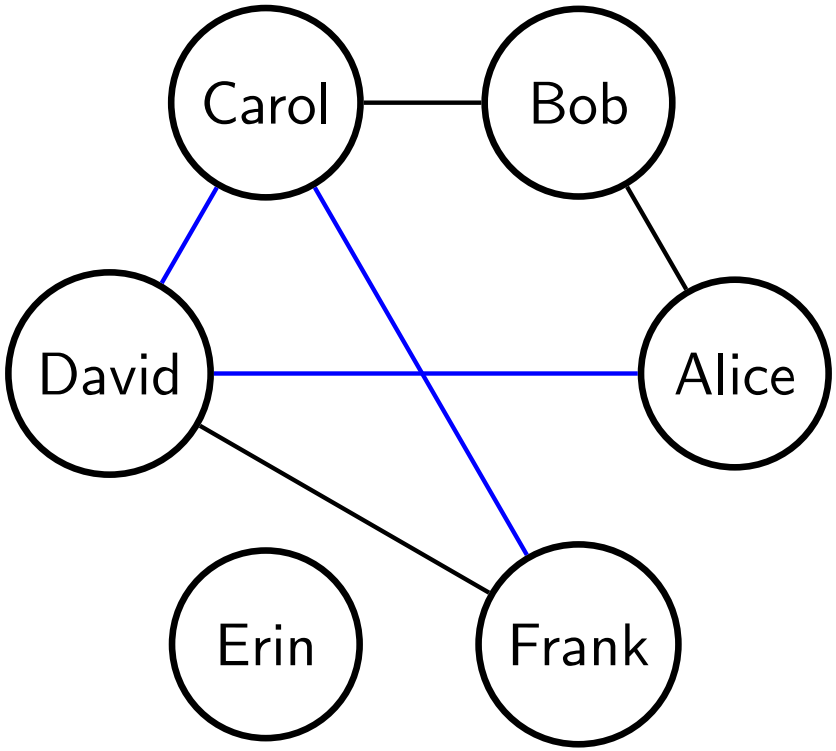

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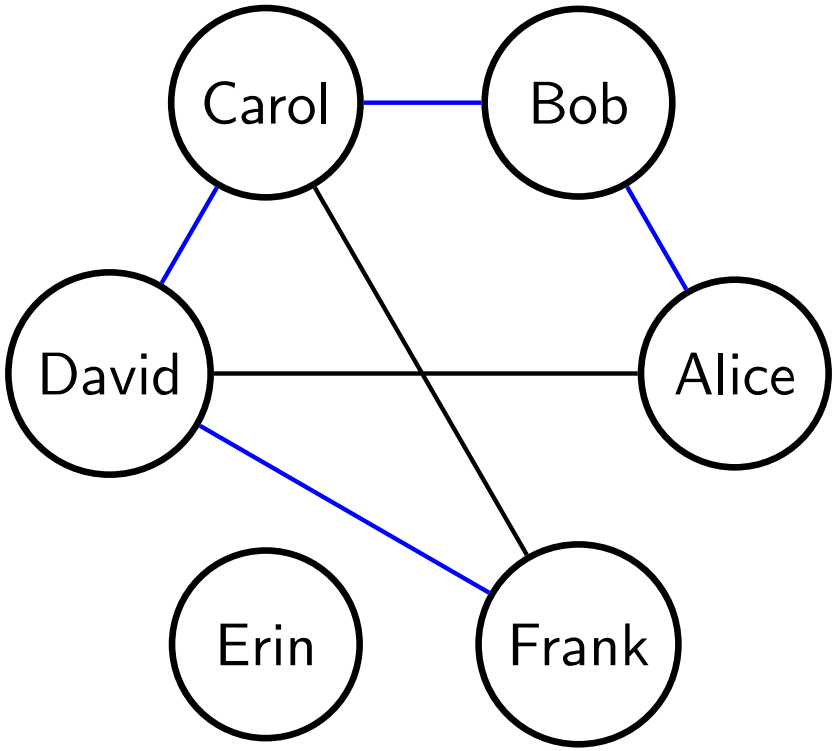

Parts of Graphs: Paths and Loops

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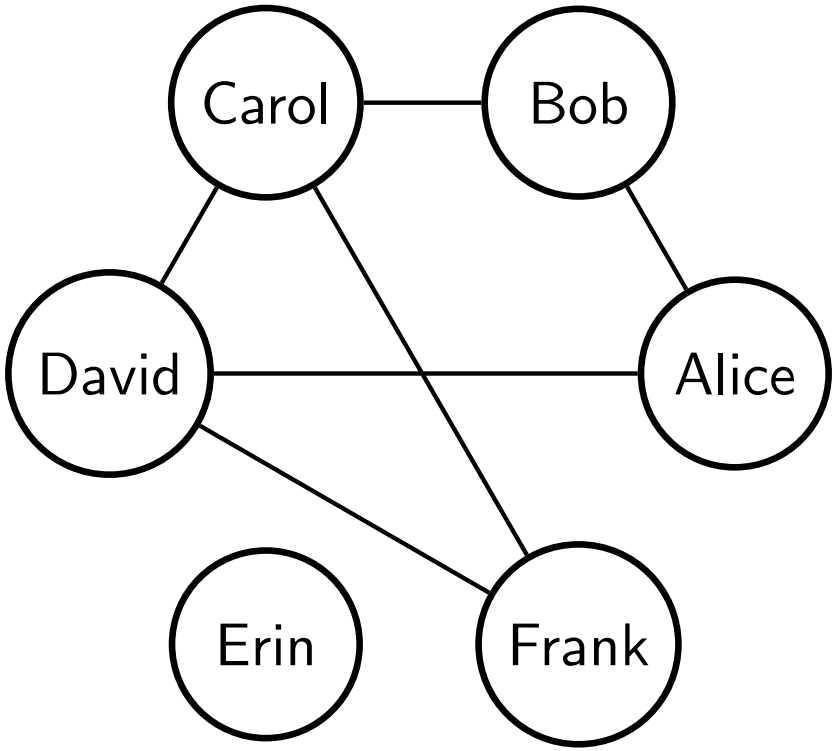

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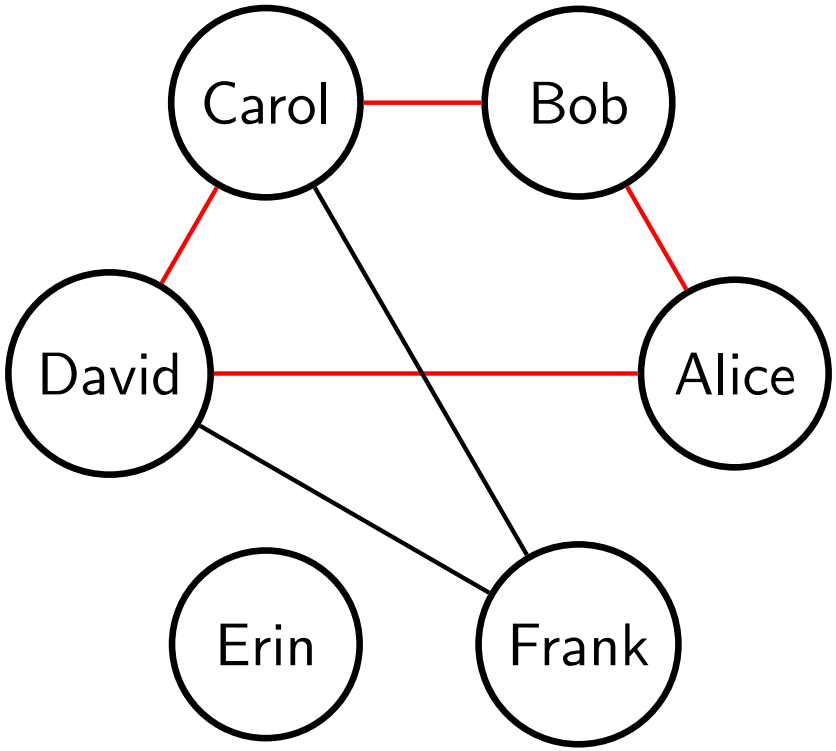

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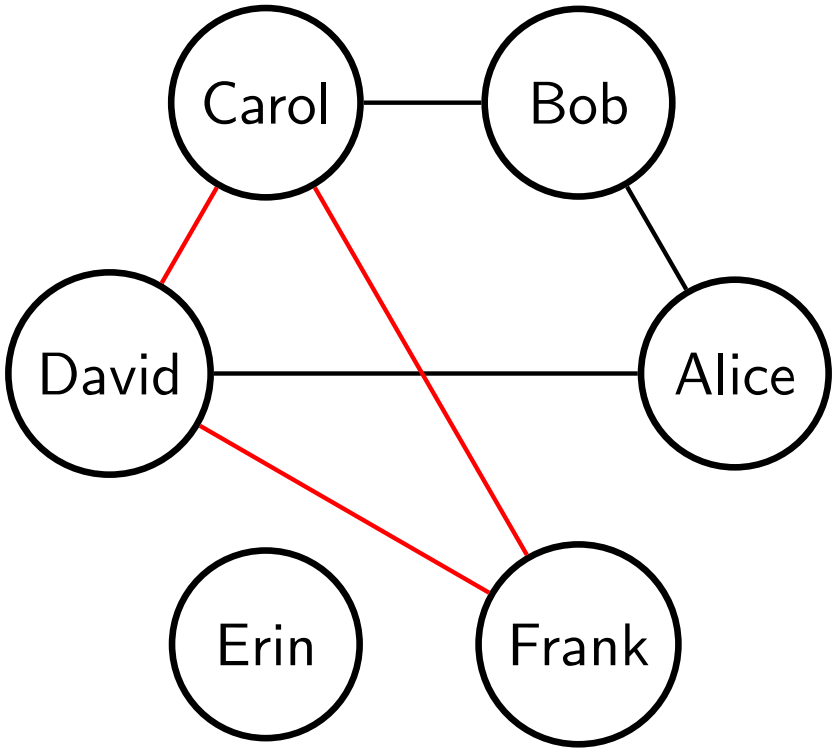

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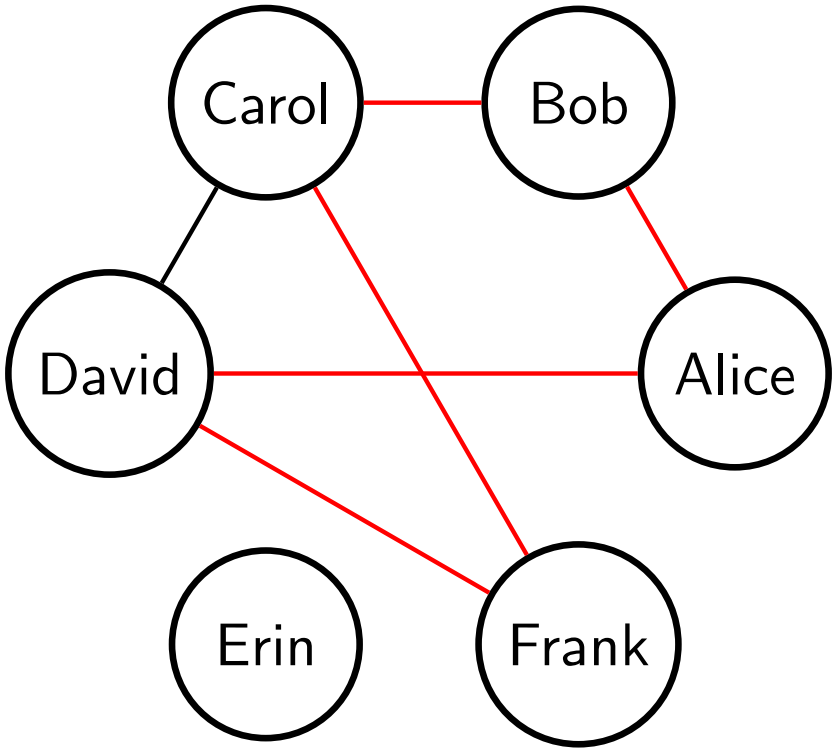

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A graph is **connected** if there is a path between any two nodes.

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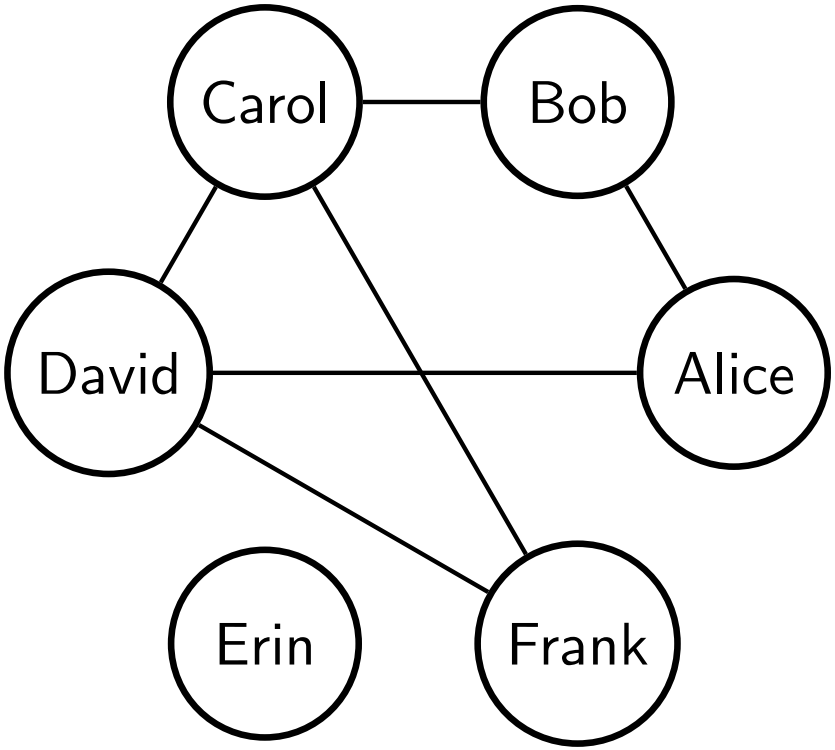

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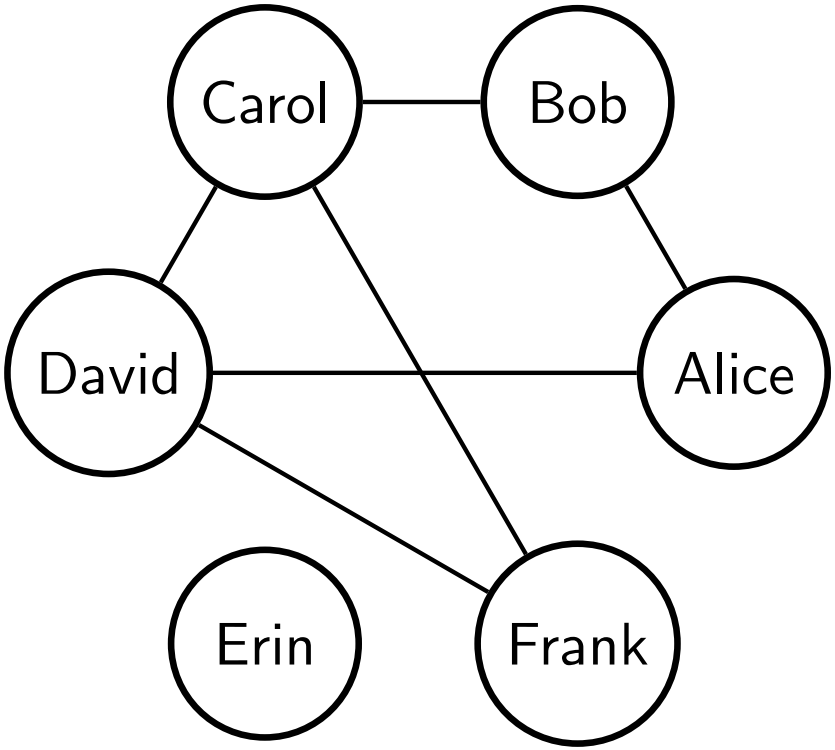

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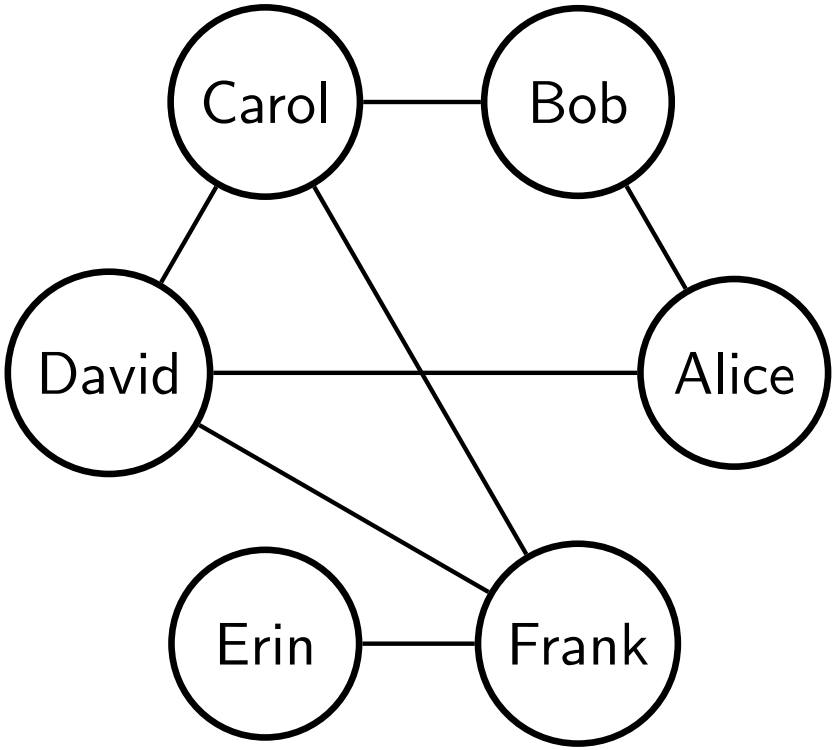
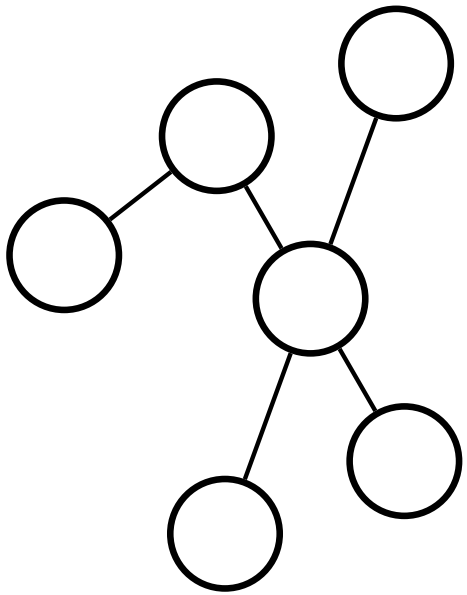
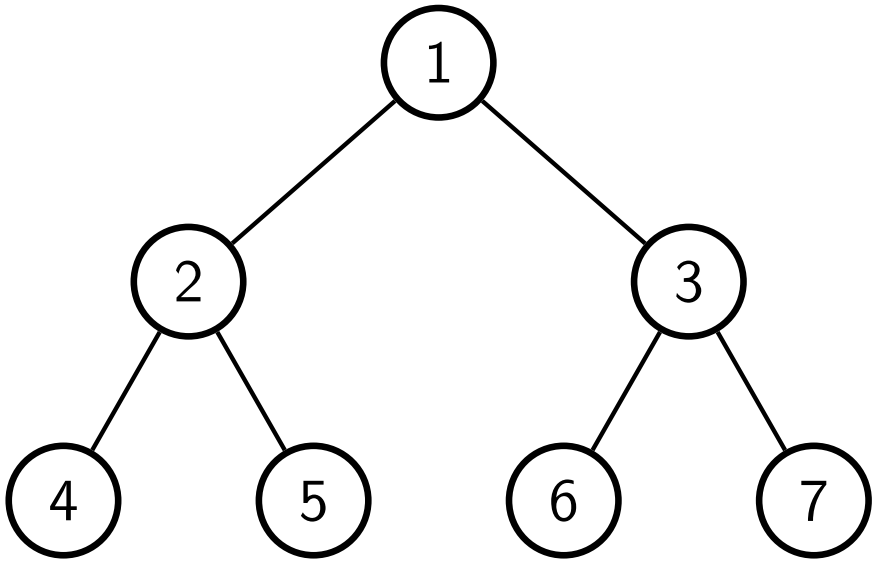


Connected!

Types of Graphs: Trees

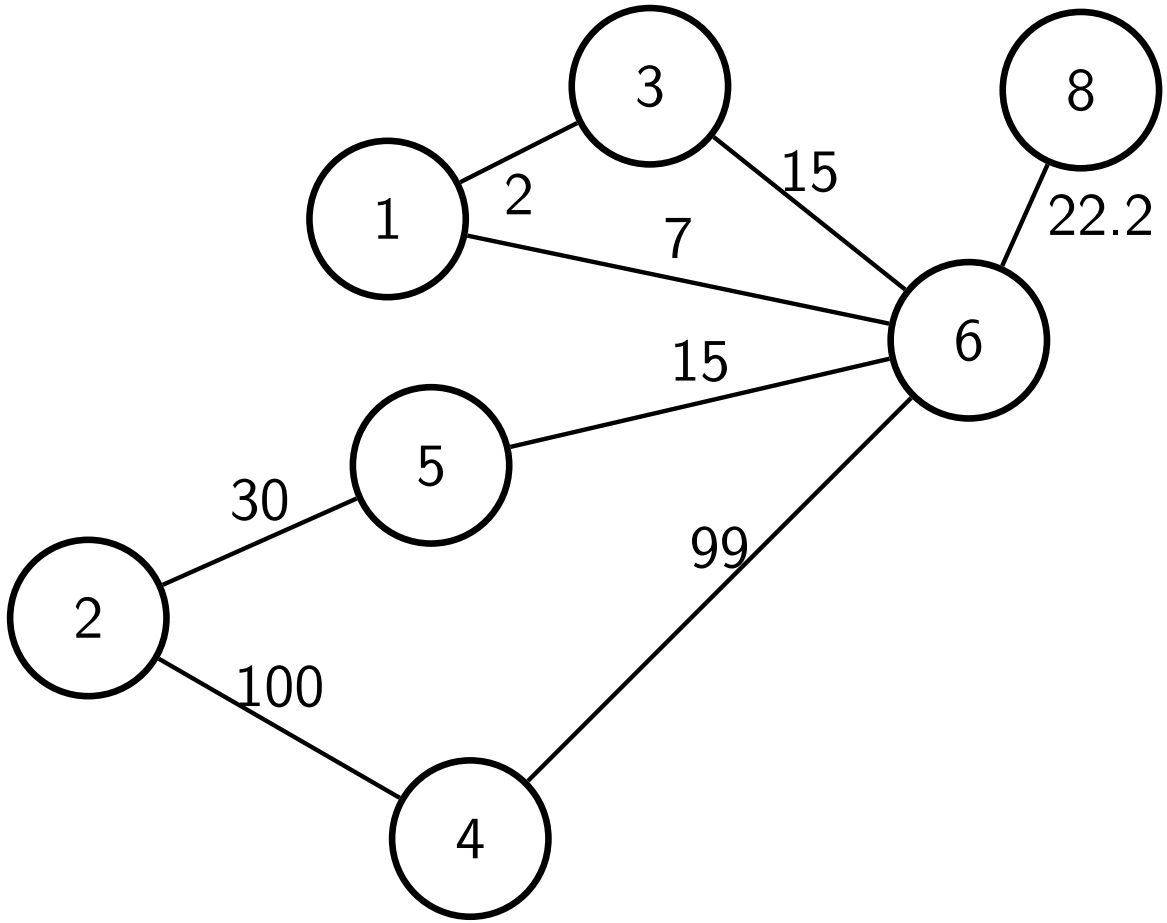
A graph is a **Tree** if there are no loops.





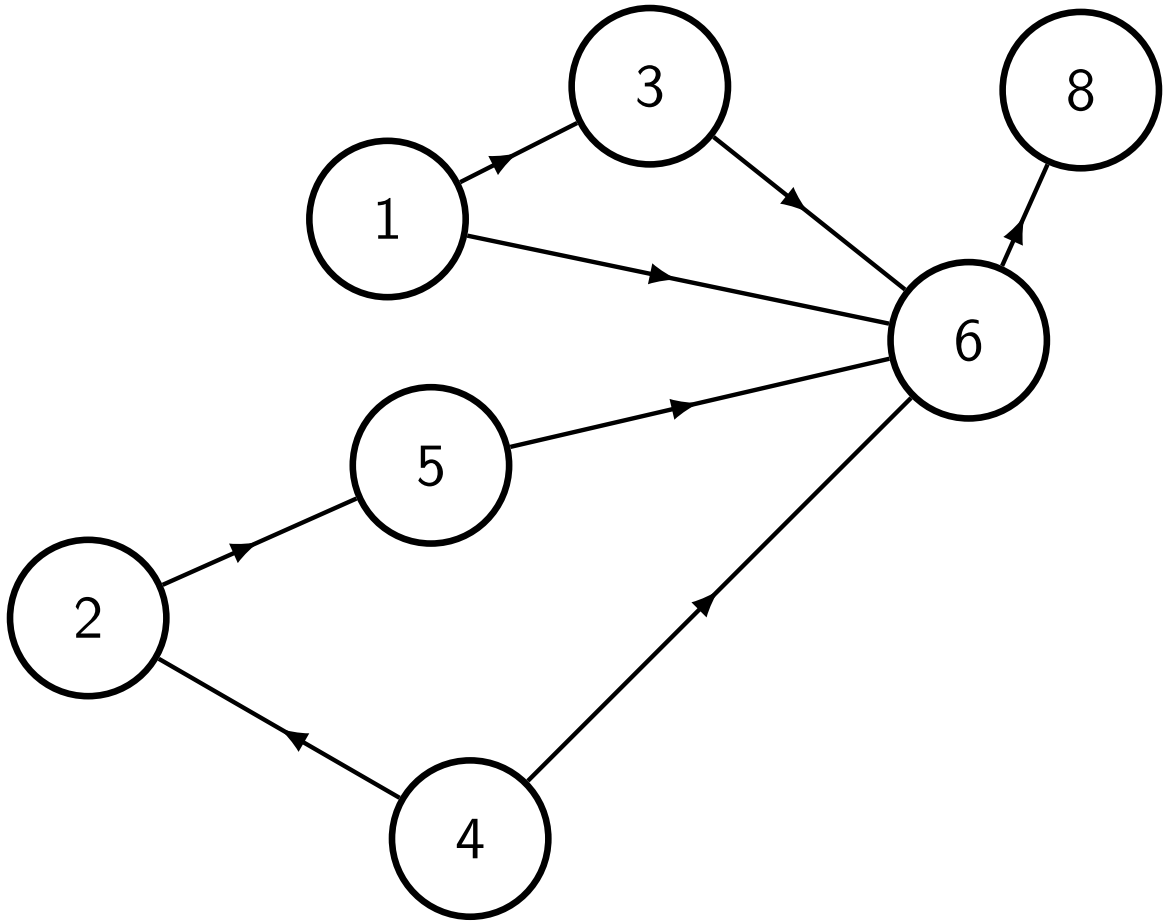
Types of Graphs: Weighted

Weighted Graph: edges have numbers with them, meaning size or length.



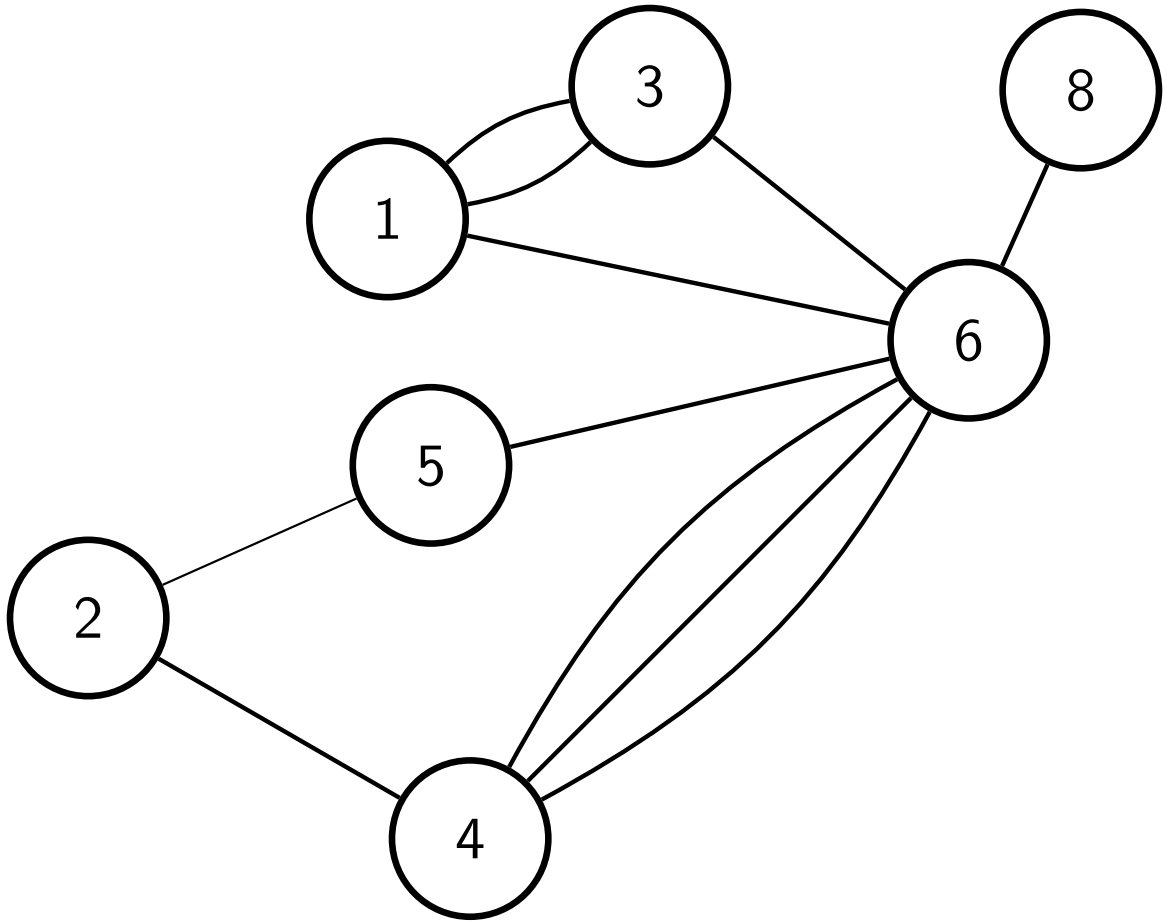
Types of Graphs: Directed

Directed Graph: edges may go in one direction.



Types of Graphs: Multigraphs

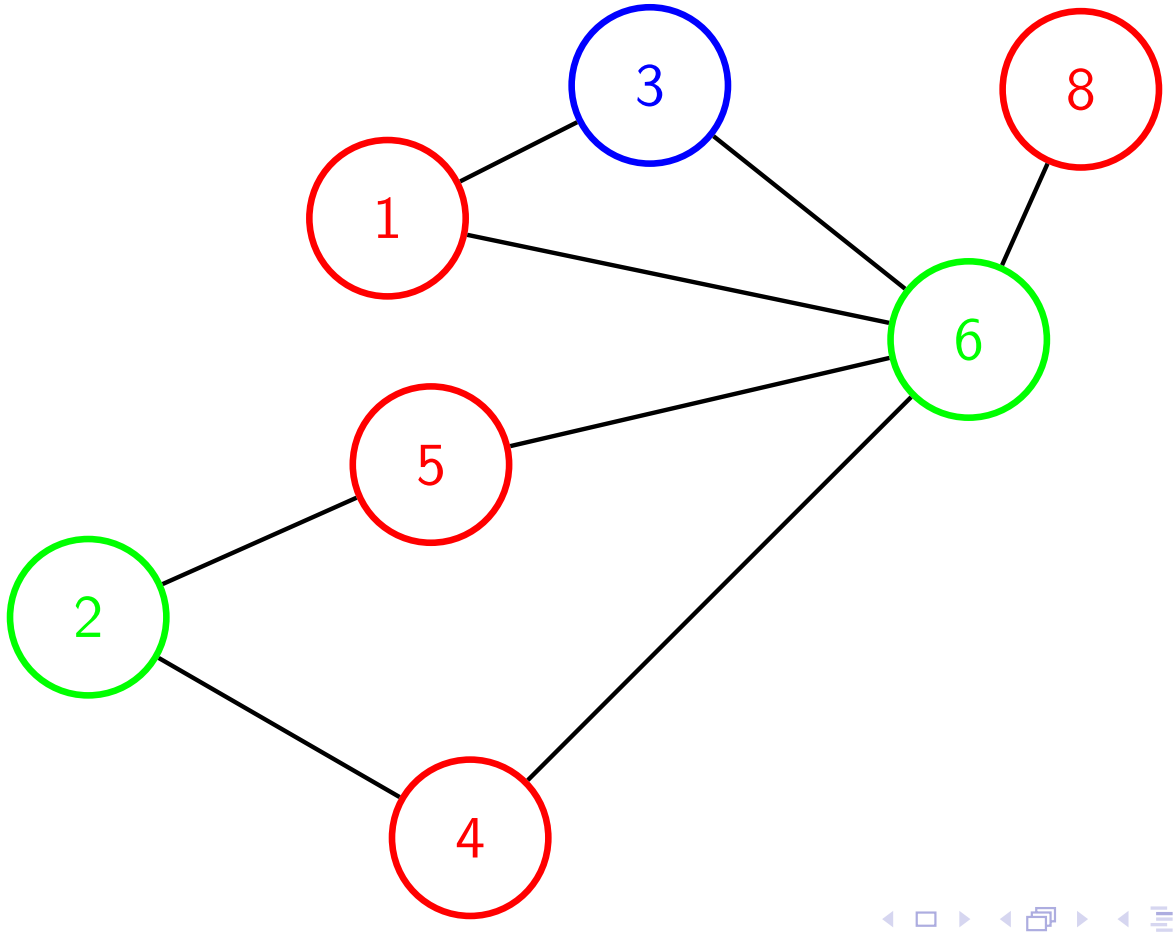
Multigraph: can have multiple edges between nodes.



Coloring Graphs

Coloring Problem: Color in each node without coloring adjacent nodes (those connected by one edge) the same color.

- This can be applied to scheduling where you need to assign a number of jobs but some of which can't be done at the same time.



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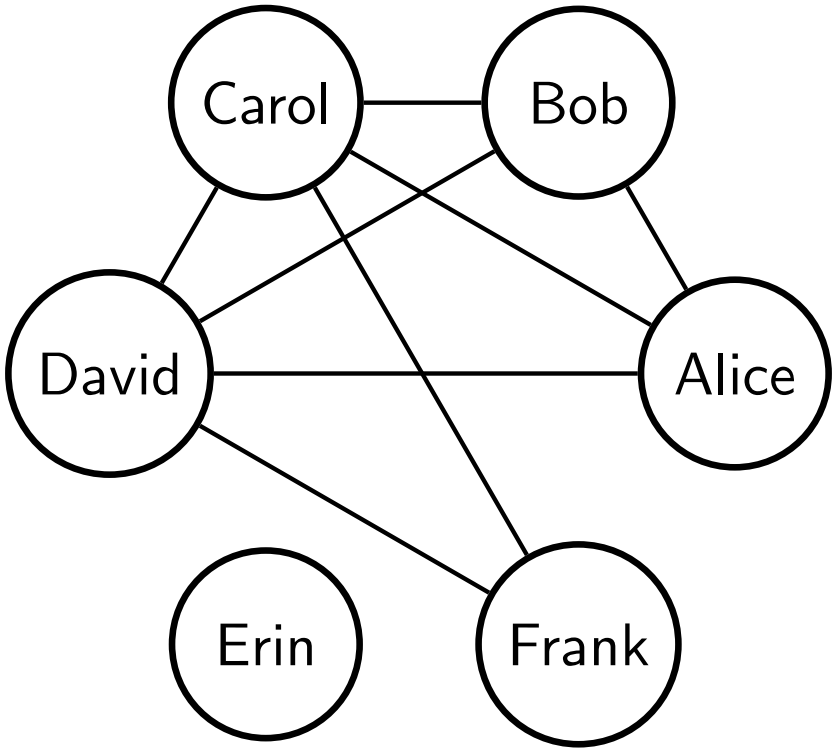
What is the smallest number of colors you need to use?



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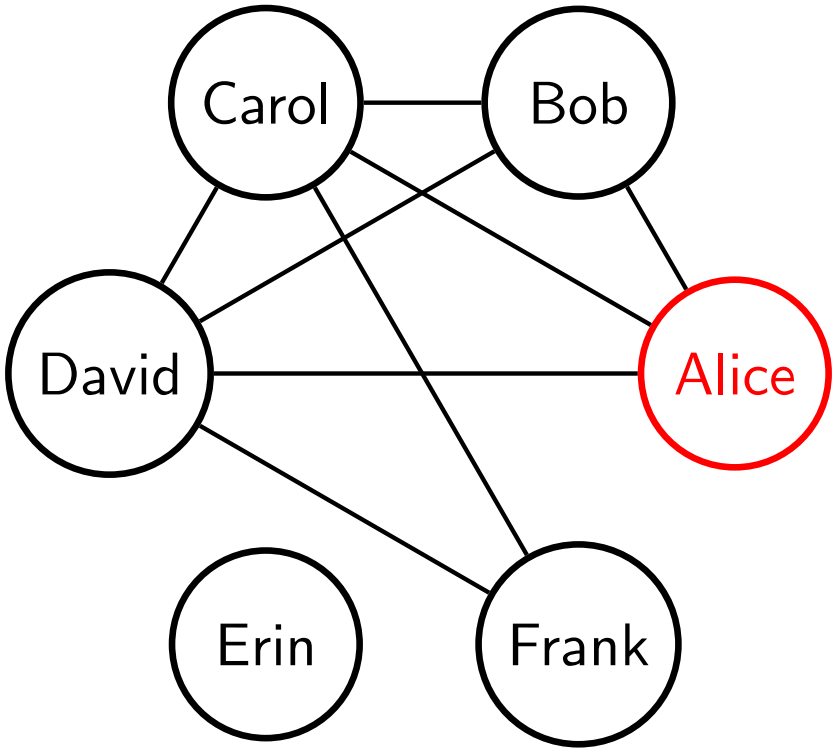

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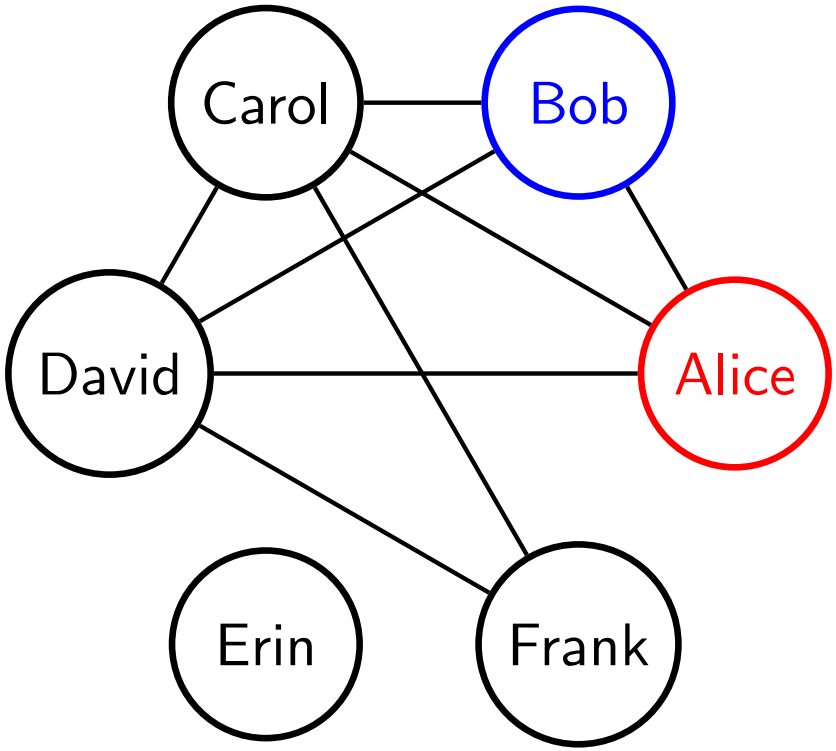

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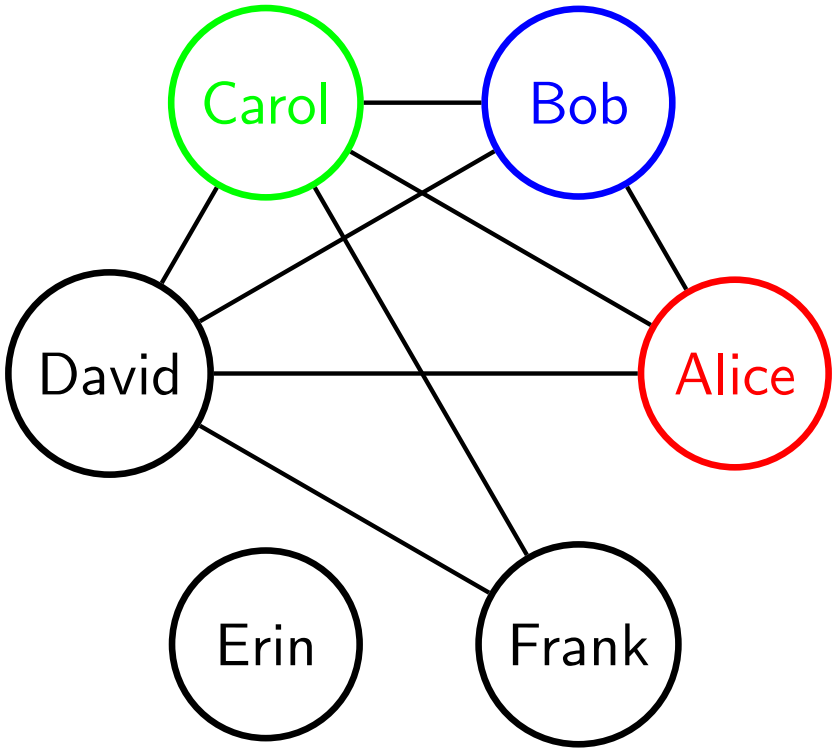

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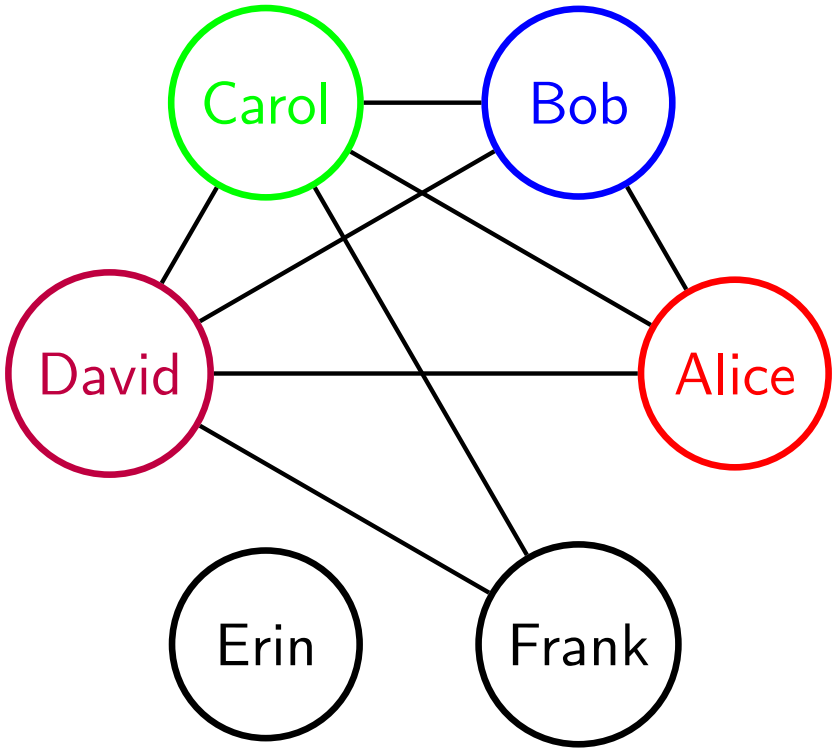

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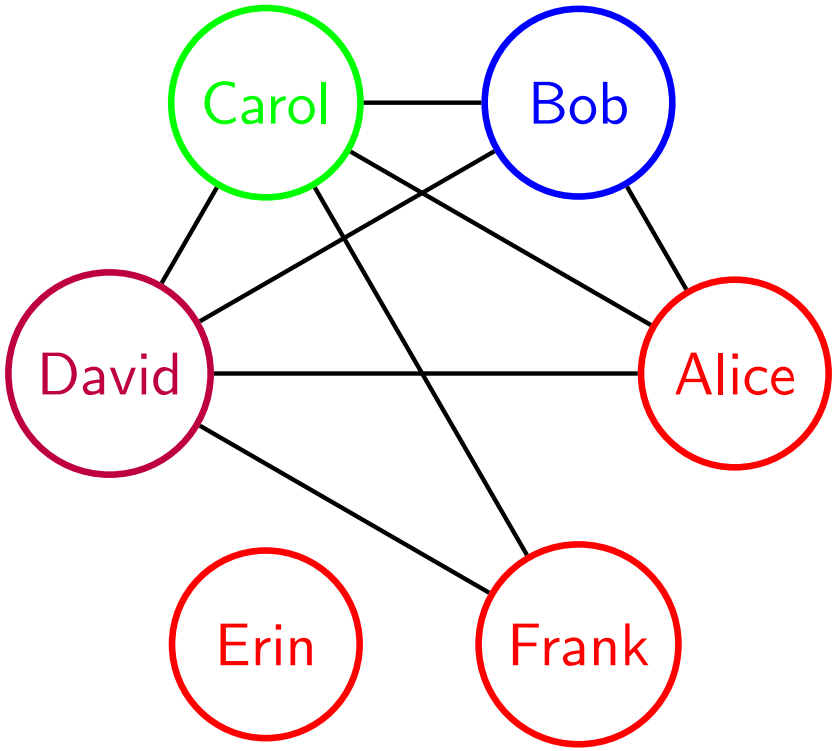

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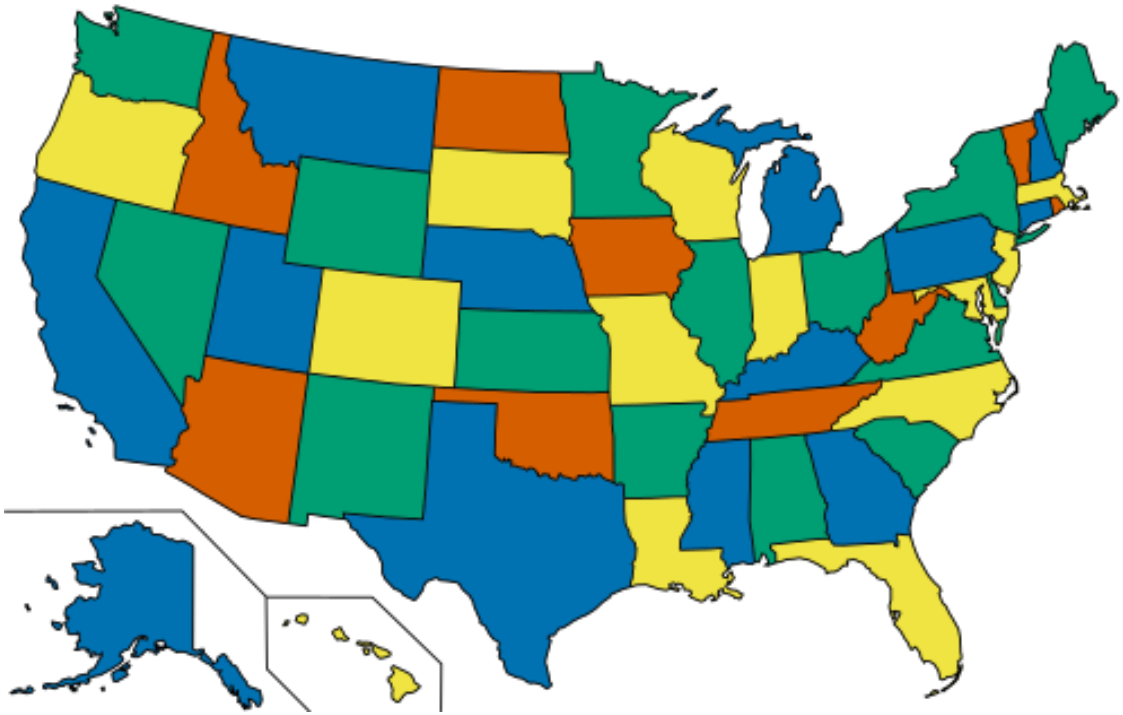

From `https://en.wikipedia.org/wiki/Four_color_theorem`