University of Maryland Girls Talk Math

Patterns and Fractals

Presented by Natalie Majerik

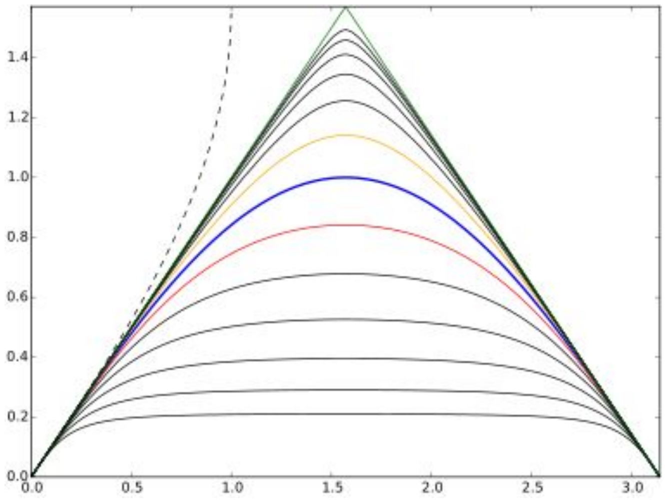
Iterative Functions

Iteration- a repetition

An iterative function repeats the same process

Uses the output as the input into the next iteration

We can iterate forward using the previous x value



Iterative Function Behavior

- Fixed points
- Divergence
- Convergence
- Bounded

Inverse Functions

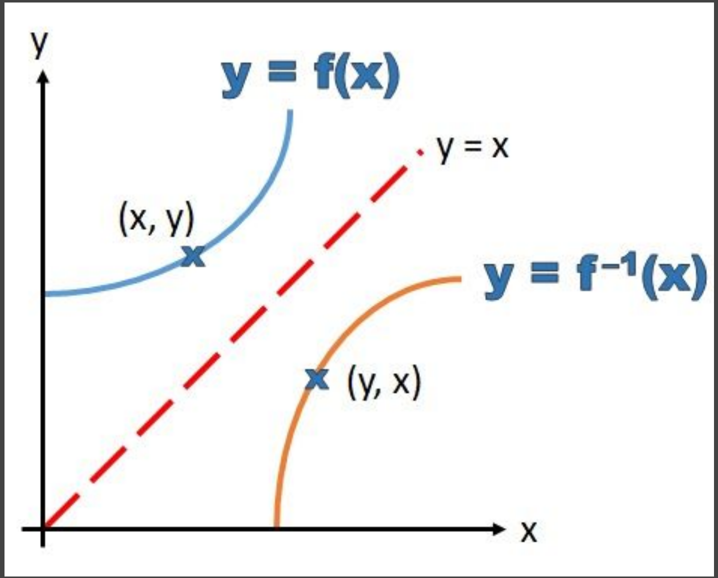
We can iterate backwards using the inverse

Inverse functions are one-to-one

- More than one x value cannot have the same y value

It reverses the function by solving for x

Function must pass the horizontal line test



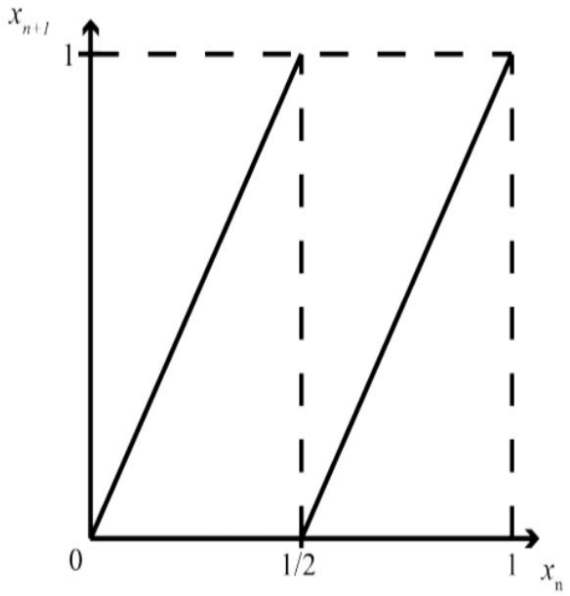
$M(x_n) = x_n - 1$ ---> $x_n = M^{-1}(x_n) - 1$ ---> $M^{-1}(x_n) = x_n + 1$

Mod Maps

Modulus functions have no inverse since more than one x value produce the same y value

With certain initial conditions, we can predict a pattern of behavior

First iteration: X_{n+1} Second iteration: X_{n+2}



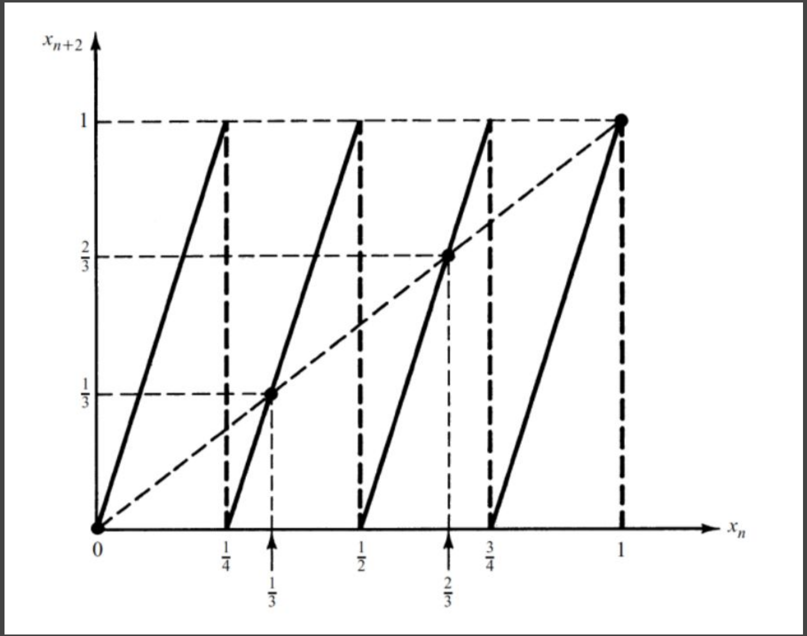
Fixed Points

There are fixed points where $x_n = x_{n+2}$

Calculate fixed points by looking where the map intersects the line

Calculate how many fixed points using

$2^p - 2$

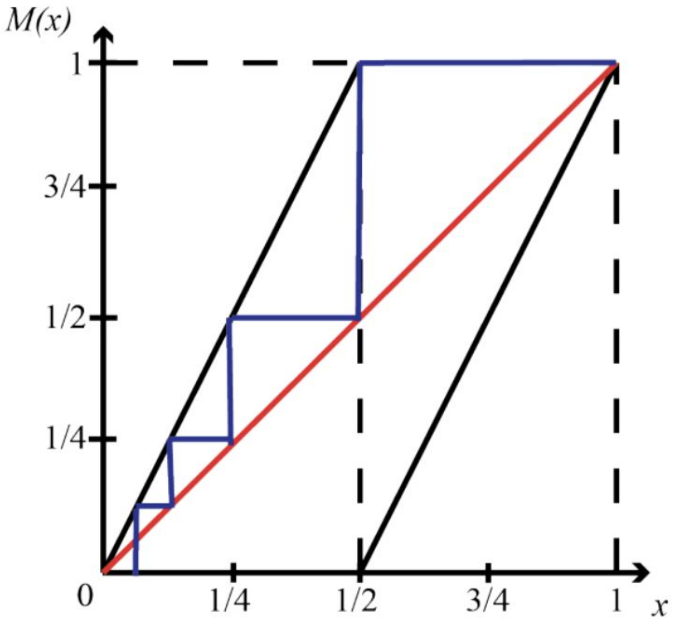


Convergence

M(x) **converges** for certain initial condition x if M(x) approaches a **constant valued fixed point**

M(x) **diverges** for certain initial conditions where M(x) approaches **positive/ negative infinity**

We can visualize convergence in Cobweb maps

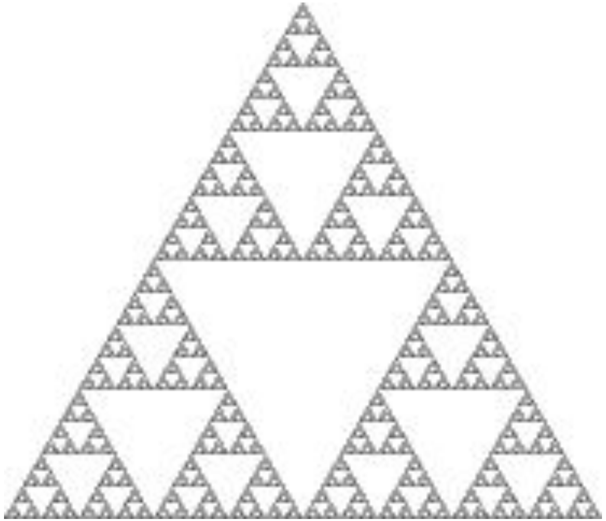


Fractals

They are self - similar

One part is the same as the whole

Can be enhanced infinitely with the proper resolution



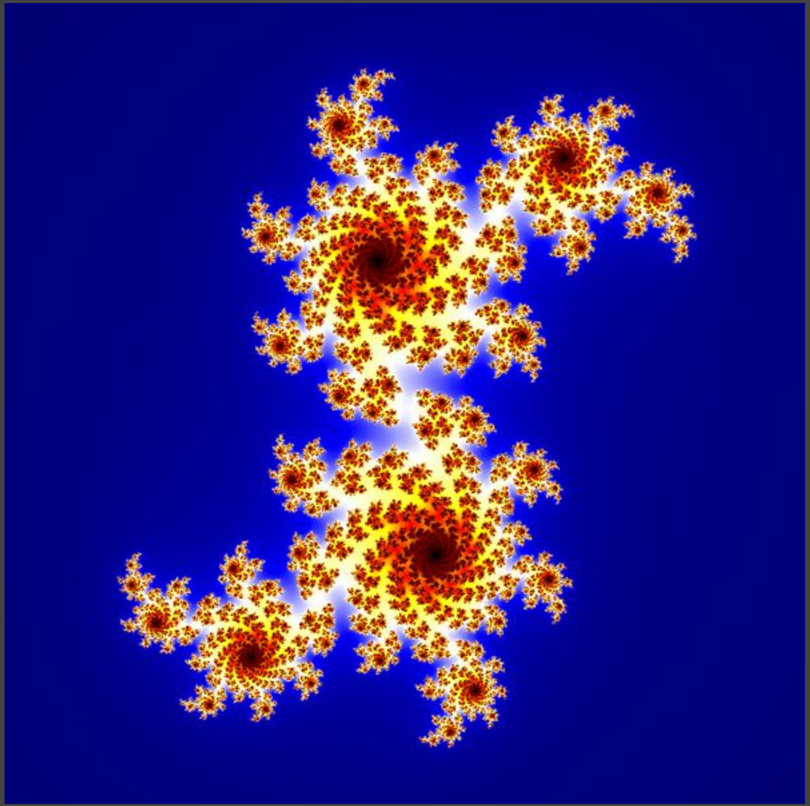
Julia Sets

When sequences stay within a specific range of numbers they are **bounded**

Mod maps are naturally bounded

log/quadratic functions require certain initial conditions to be bounded

For the following fractals, we will use complex numbers



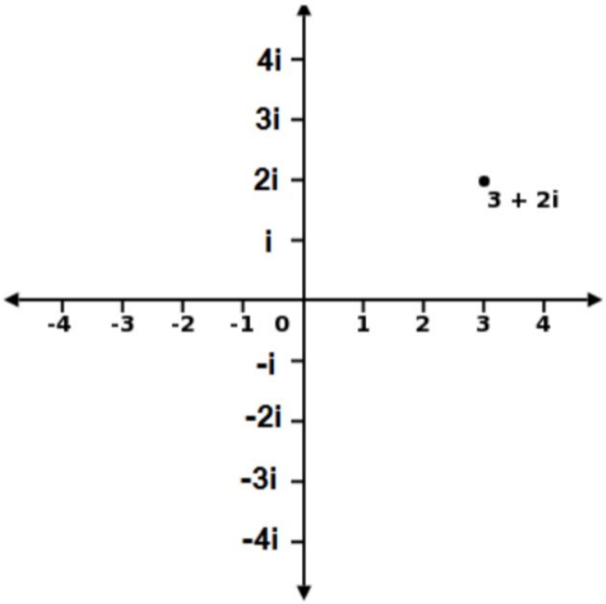
Complex Numbers

Can you take the square root of a negative number?

What is the imaginary number *i*?

$3 + \sqrt{-4}$

$(2+3i)(4+i)$

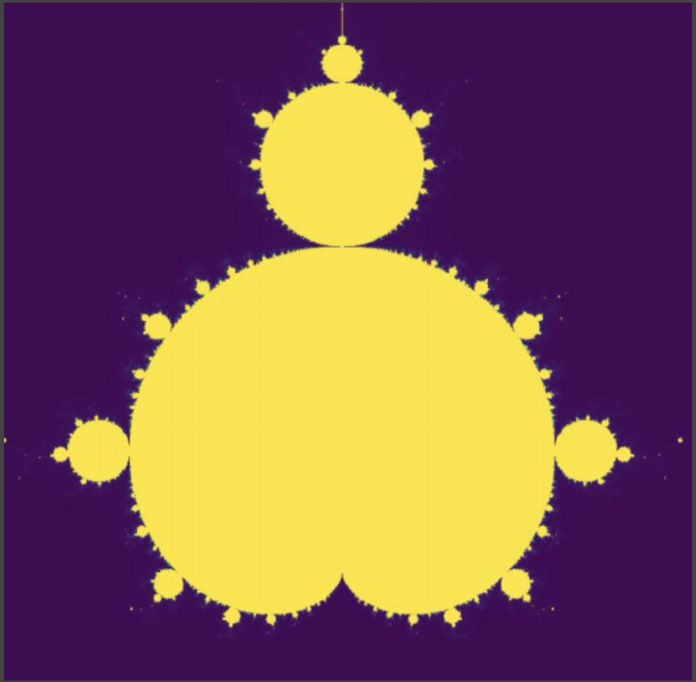


Mandelbrot Sets

Using the equation, we vary the “c” value

For a particular c, if the orbit stays bounded under iteration, c is in the **Mandelbrot set**

If it diverges, it is not in the set



$$f(z) = z^2 + c$$

Kahoot Time!
Test your knowledge

Game pin:

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