Number Systems and Scientific Computing

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Outline

- **What is a number system?**
- Examples of number systems
  - Decimal
  - Binary
  - Hexadecimal
  - Mayan numerals
- How are number systems used in scientific computing?

Many ways to represent the same number

Decimal form
45

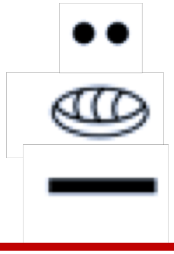
Hexadecimal form
$2D_{16}$

Binary form
101101_2

Octal form
55_8

Roman numerals
XLV

Mayan numerals



Many ways to represent the same number

Decimal form
45

Hexadecimal form
2D

Binary form
101$_2$

Why do we need different number systems?

How do we convert between them?

Roman numerals
XLV

Mayan numerals

What is a number system?

- A *number system* is a standard for representing numbers in written form or for computation
- Examples
  - Decimal form (345.01)
  - Roman numerals (MDCCXXXII)
  - Binary form (011001)

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- Examples
  - Decimal form (345.01)
  - Roman numerals (MDCCXXXII)
  - Binary form (011001)

Why are number systems important?

- The number system dictates how many symbols it takes to communicate a number
- Some systems are more useful for communicating among humans, while others are more useful for communicating with computers

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  - Binary
  - Hexadecimal
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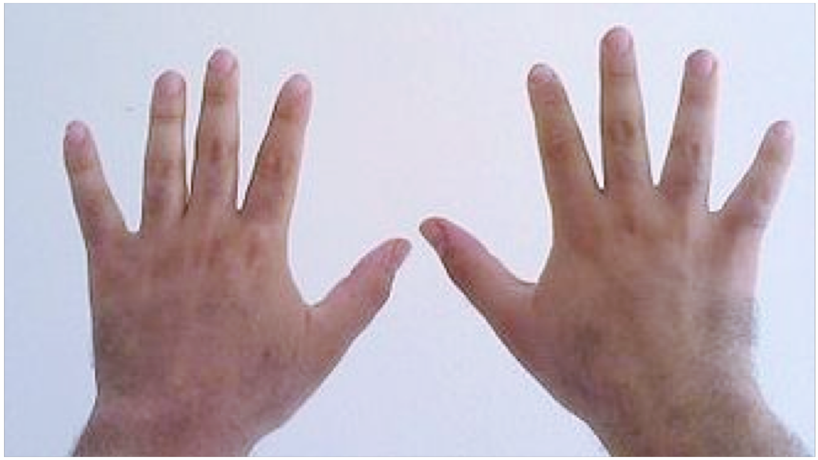
- How are number systems used in scientific computing?

How does the decimal system work?

- Number system based on 10 digits (base 10)
- Most common way to represent a number for arithmetic



10 possible digits:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

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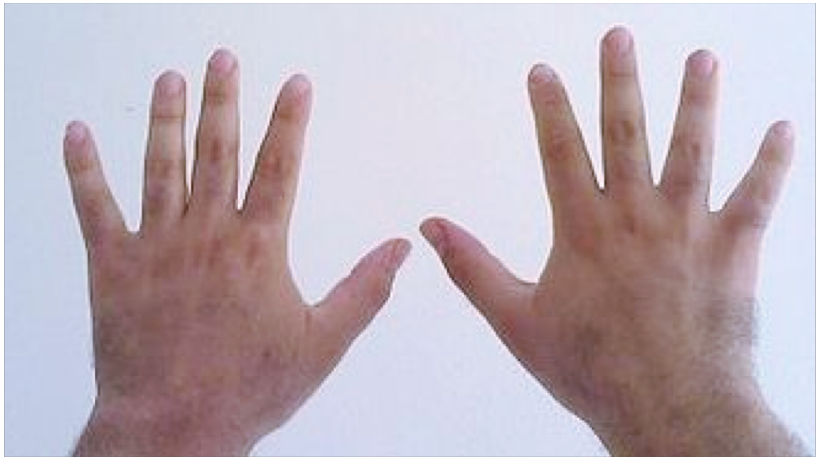

10 possible digits:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Decimal is a positional system

- The weight of each digit given by its position with respect to decimal point
- The “0” becomes valuable in the decimal system!

Let’s dissect a decimal number

5	8	3	9	.	0	0	3	4

Decimal point

Let's dissect a decimal number

5	8	3	9	.	0	0	3	4
10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	10^{-4}

$$5839.0034 = 5\times10^{3} + 8\times10^{2} + 3\times10^{1} + 9\times10^{0} + 0\times10^{-1} + 0\times10^{-2} + 3\times10^{-3} + 4\times10^{-4}$$

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*Position with respect to decimal indicates powers of **10***

Binary number systems

- The binary system is similar to the decimal system
  - Positional system based on powers of 2
- Two possible digits (0 and 1), or *bits*

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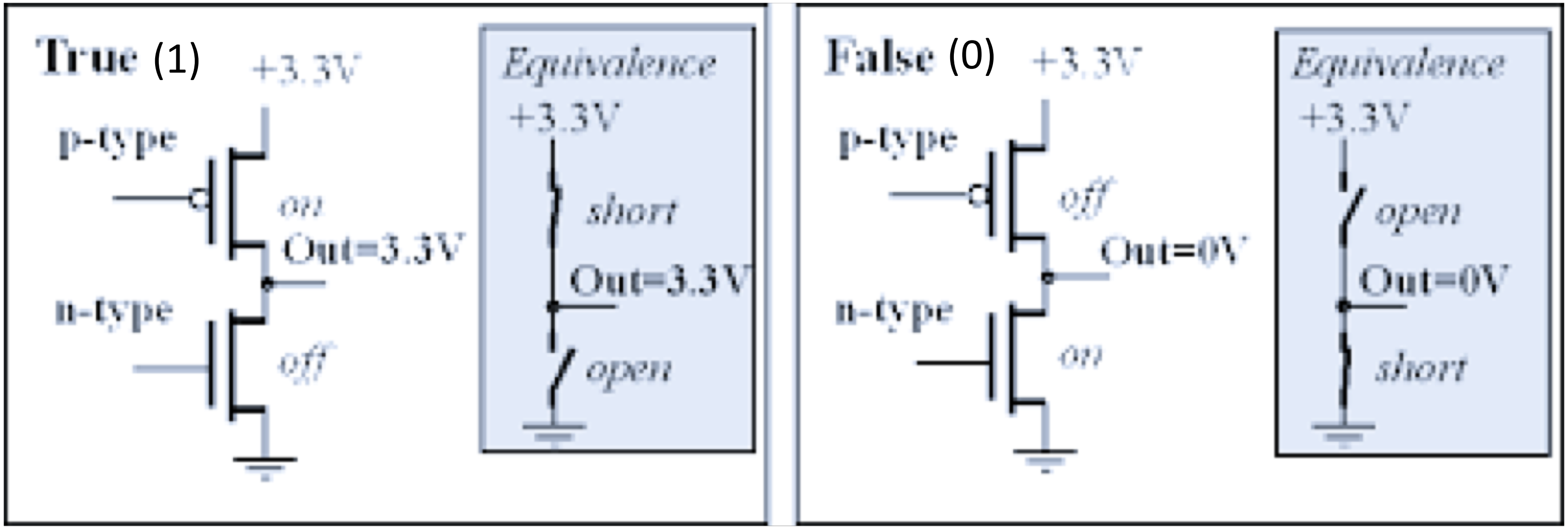
Why is binary important?

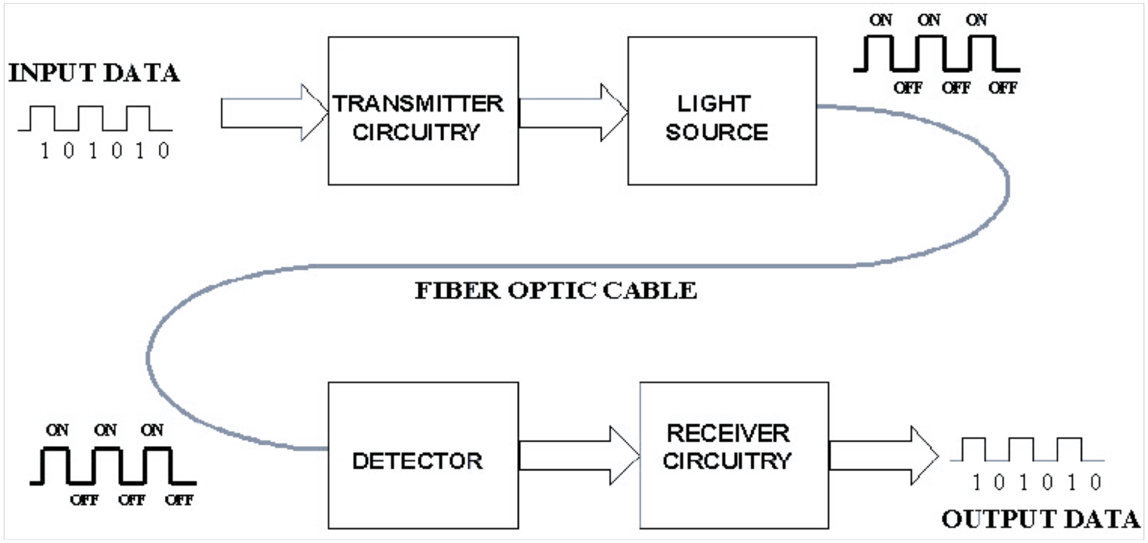
  - Easy to represent electronically
  - Only 2 states needed to store a given digit (e.g. on and off)
  - Used in almost all modern computers
  - Basis for Boolean data
    - 0 = False, 1 = True

How do computers use binary?

- Transistor switches are the building blocks of computers
- Can be fundamentally in two states: on and off
- Each stores one byte (digit of binary) of information

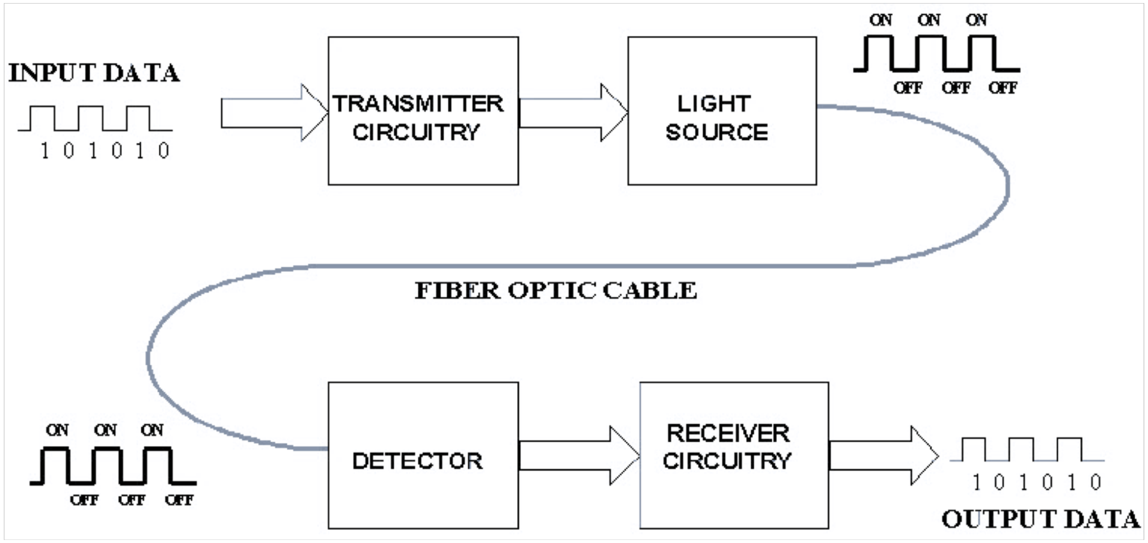


Fiber optics



- Used to transmit telephone signals, internet connections, and cable television

Fiber optics



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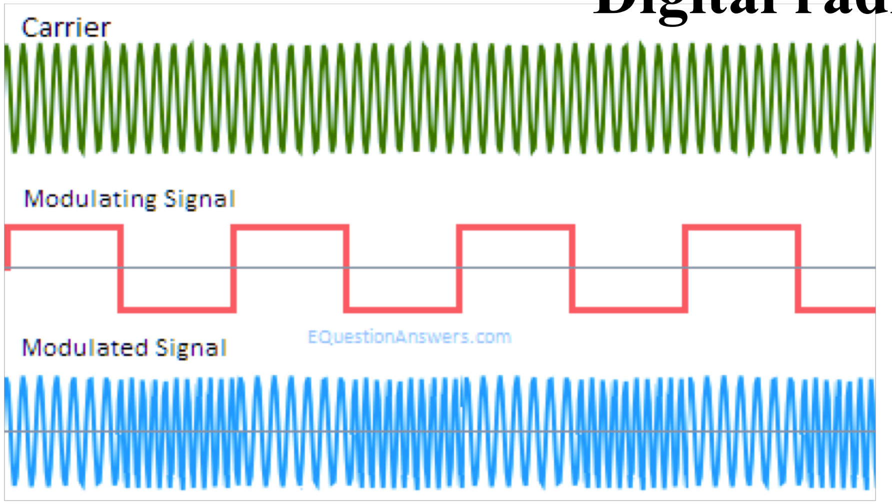
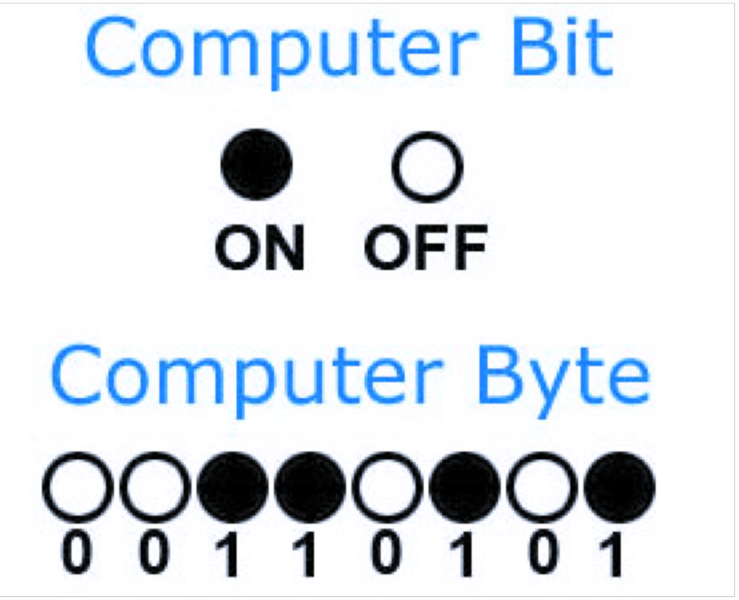
Digital radio signals



- Frequency and amplitude modulation
- Wi-fi transmission
- Digital TV

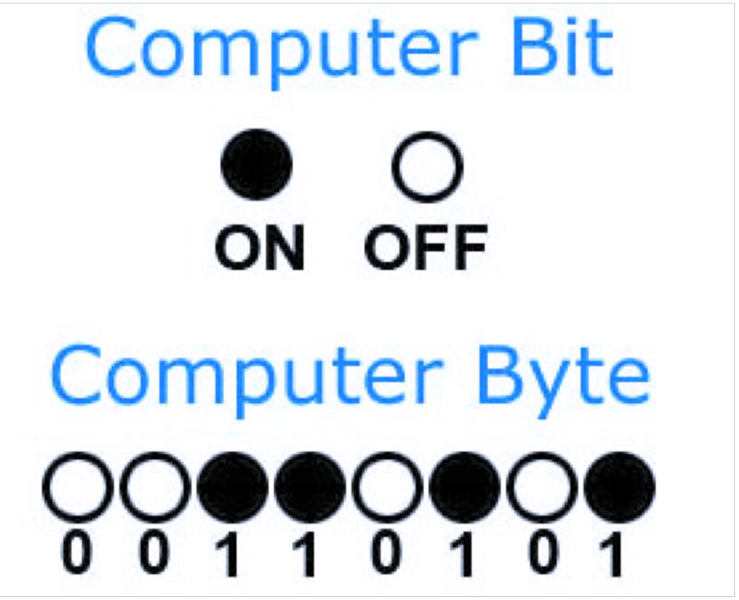
Bits, bytes, and gigabytes



- Units of measurement for digital memory and transmission
- How many binary digits needed to store a piece of data (i.e. music file, photo)

Byte (B)	$8 = 2^3$ bits
Kilobyte (KB)	$1024 = 2^{10}$ bytes
Megabyte (MB)	1024 kilobytes
Gigabyte (GB)	1024 megabytes
Terabyte (TB)	1024 gigabytes
Petabyte (PB)	1024 terabytes

Bits, bytes, and gigabytes



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Com

0

Comp

0 0 1 1 0 1 0 1

How many binary digits can a 16 GB flash drive store?

to store

hoto)

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Bits, bytes, and gigabytes

Com

Comp

0 0 1 1 0 1 0 1

How many binary digits can a 16 GB flash drive store?

$16 \times 1024^3 \times 8 = 137{,}438{,}953{,}472$

to store
hoto)

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Let's dissect a binary number

1	0	1	0	.	1	1	0	1

↑

Decimal point

Let's dissect a binary number

1	0	1	0	.	1	1	0	1
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}

$$1010.1101_2 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 + 1\times2^{-1} + 1\times2^{-2} + 0\times2^{-3} + 1\times2^{-4}$$

$$= 8 + 0 + 2 + 0 + ½ + 1/4 + 0 + 1/16$$

$$= 10.8125_{10}$$

Position with respect to decimal indicates powers of ***2***

Now it's your turn

Convert the following to decimal form

1100.0_2

"Base 2"

Convert the following to decimal form

1100.0_2

1	1	0	0	.	0
2^3	2^2	2^1	2^0		2^{-1}

$$1100.0_2 = 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 0\times 2^0 + 0\times 2^{-1}$$

$$= 8 + 4 + 0 + 0 + 0$$

$$= 12.0_{10}$$

"Base 10"

Going the other direction

Convert the following decimal to binary form

$$75_{10}$$

Going the other direction

Convert the following decimal to binary form

$$75_{10}$$

Successively divide by 2, keeping track of the remainder

$75/2 = 37 + 1/2$
$37/2 = 18 + 1/2$
$18/2 = 9 \quad + 0/2$
$9/2 \quad = 4 \quad + 1/2$
$4/2 \quad = 2 \quad + 0/2$
$2/2 \quad = 1 \quad + 0/2$
$1/2 \quad = 0 \quad + 1/2$

Going the other direction

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$4/2 = 2 + 0/2$
$2/2 = 1 + 0/2$
$1/2 = 0 + 1/2$

Use each remainder, beginning at bottom

$$75_{10} = 1001011_2$$

Hexadecimal system

- The binary system requires many digits to represent a small number (i.e. $12_{10} = 1100_2$)
- To make it easier to handle by humans, a hexadecimal (base 16) system is sometimes used

16 possible digits:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

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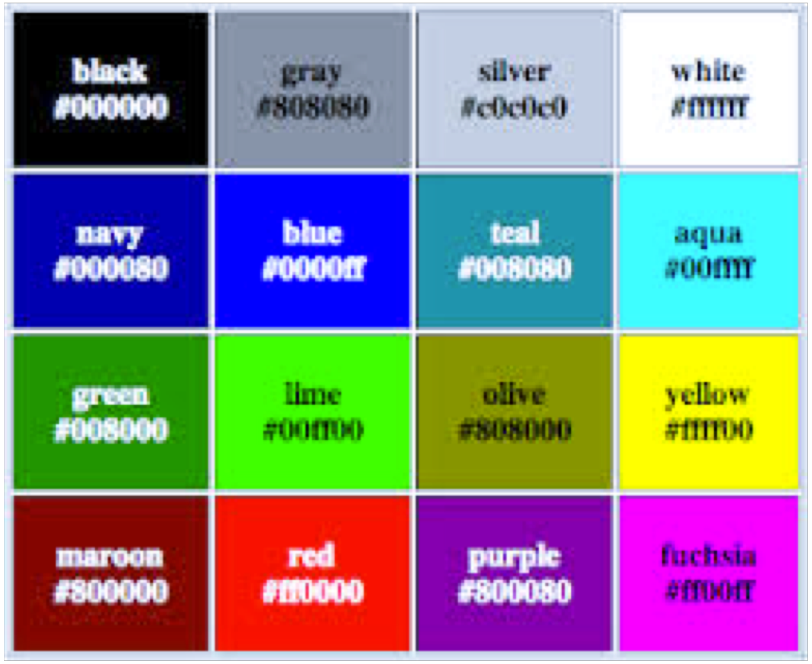
Applications of hexadecimal

- Used to define colors in HTML/CSS languages
  - #RRGGBB
- Defining numbers in assembly language (i.e. locations in memory)



Hexadecimal form can easily be converted to binary

0_{16}
1_{16}
2_{16}
3_{16}
4_{16}
5_{16}
6_{16}
7_{16}
8_{16}
9_{16}
A_{16}
B_{16}
C_{16}
D_{16}
E_{16}
F_{16}

→

0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Hexadecimal form can easily be converted to binary

0_{16}	0	0	0	0
1_{16}	0	0	0	1
2_{16}	0	0	1	0
3_{16}	0	0	1	1
4_{16}	0	1	0	0
5_{16}	0	1	0	1
6_{16}	0	1	1	0
7_{16}	0	1	1	1
8_{16}	1	0	0	0
9_{16}	1	0	0	1
A_{16}	1	0	1	0
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Once conversion of base digits known, larger numbers can be built up

$$A93_{16} = 101010010011_2$$

Hexadecimal form can easily be converted to binary

0_{16}	0	0	0	0
1_{16}	0	0	0	1
2_{16}	0	0	1	0
3_{16}	0	0	1	1
4_{16}	0	1	0	0
5_{16}	0	1	0	1
6_{16}	0	1	1	0
7_{16}	0	1	1	1
8_{16}	1	0	0	0
9_{16}	1	0	0	1
A_{16}	1	0	1	0
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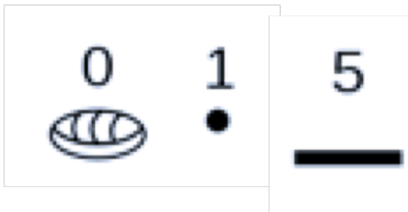
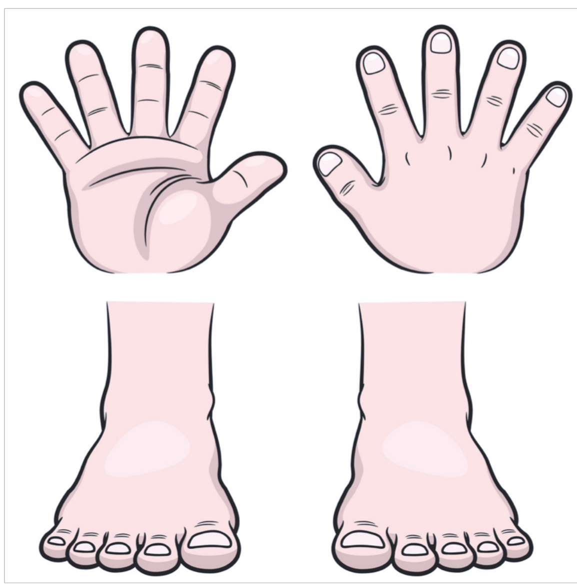
Hexadecimal is a "short form" for binary

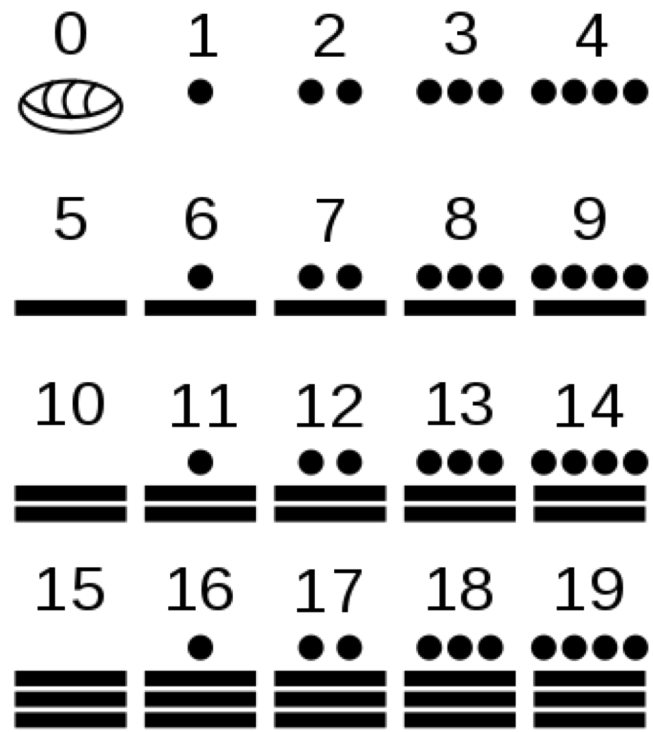
Number systems throughout history – Mayan system

- Base-20 system
- Positional system for positive integers
- 3 base symbols

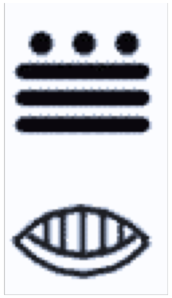




All other symbols (0-19) built up from base



- Vertical position indicates value

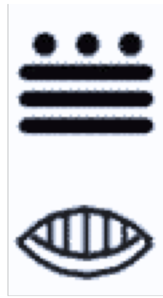


18×20^1

0×20^0

360

- Vertical position indicates value

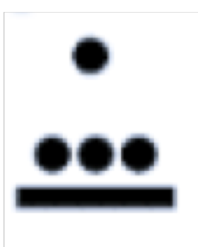


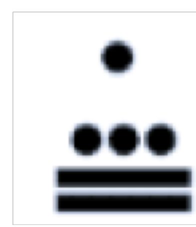
18×20^1

0×20^0

360

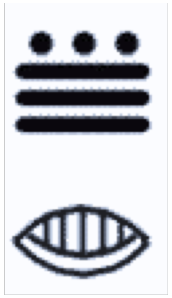
- Let's try a few more





Building up Mayan numbers

- Vertical position indicates value

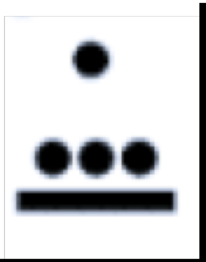


18×20^1

0×20^0

1960

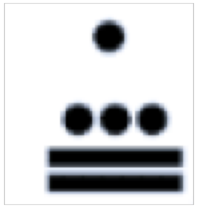
- Let's try a few more



1×20^1

8×20^0

28



1×20^1

13×20^0

33

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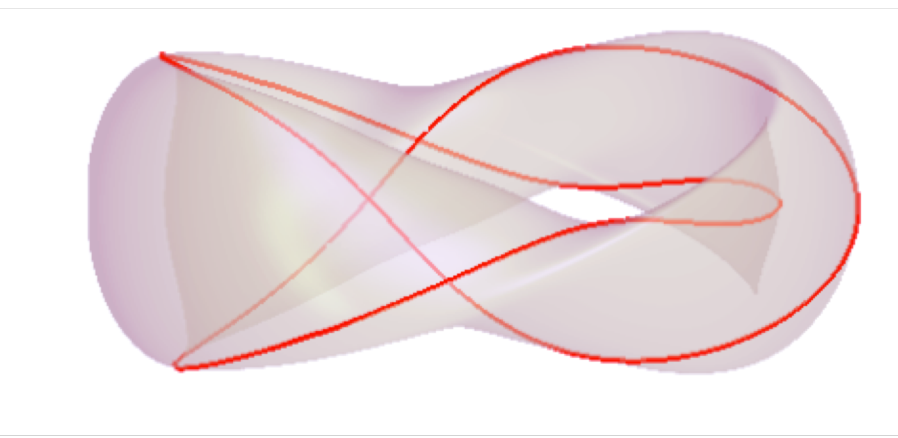
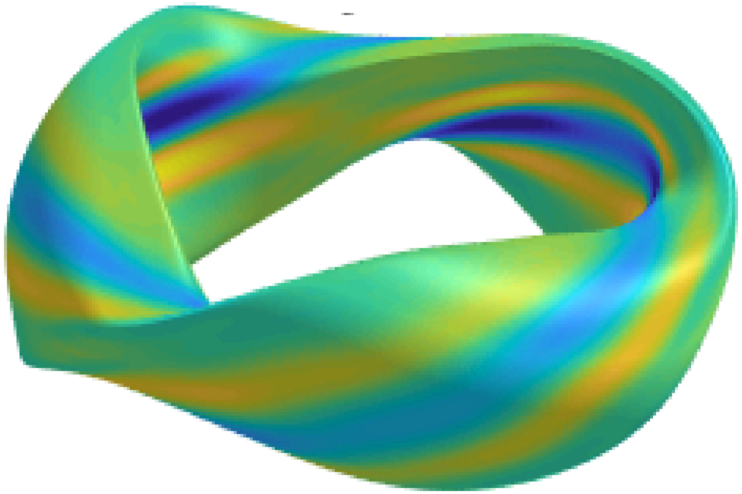
  - Hexadecimal
  - Mayan numerals
- **How are number systems used in scientific computing?**

What is scientific computing?

- *Scientific computing* is the use of computers for solving mathematical and scientific problems
- Efficient algorithms for development of numerical tools
- Used across many scientific disciplines
  - Evolution of universe
  - Molecular dynamics
  - Plasma magnetic confinement





Number systems in scientific computing

- We need to be able to deal with negative numbers and with very small/large values
  - i.e -3.45×10^{-15}, 4.89×10^{13}
- Floating point numbers, commonly used in programming languages, uses a 32 bit binary system

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Floating point representation: 01000001110111001000100000000000

Bit number	Size	Name
31	1 bit	Sign (S)
23-30	8 bits	Exponent (E)
0-22	23 bits	Mantissa (M)

Decimal representation: $(-1)^{S}(2)^{E-127}(1+M) = 27.56640625$

Conclusions

- Number systems allow us to communicate numbers with each other and with the digital world
- Several examples
  - Decimal – useful for human understanding
  - Binary – useful for digital communication
  - Hexadecimal – easily converts to binary, human readable
- Important for solving equations on computers for understanding the physical world

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Thank you for your attention!