

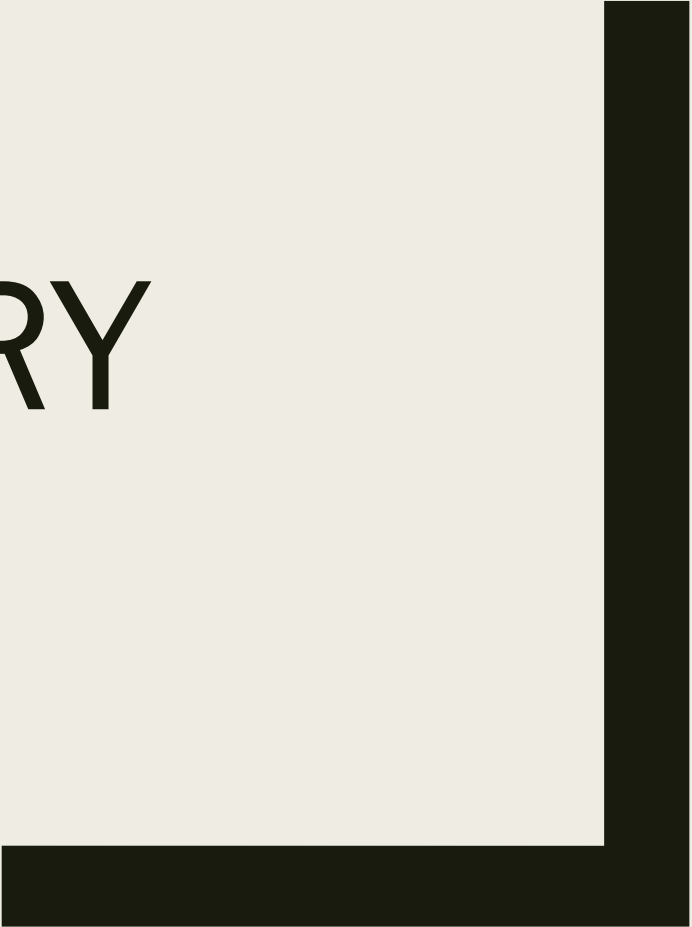


KNOT THEORY

Sarah Burnett

Girls Talk Math

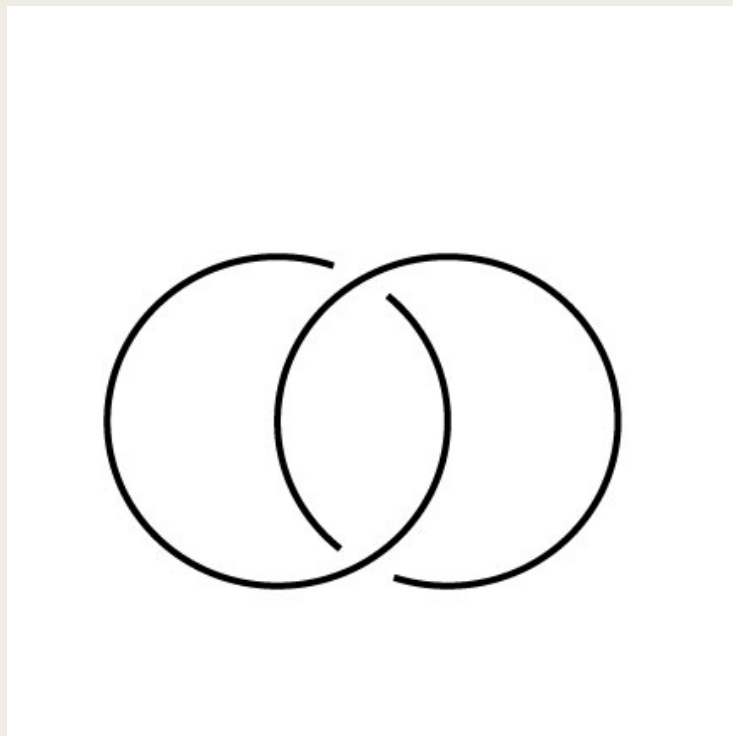
7/16/2018



Motivation

- String Theory – beginning of the universe
- Topology – how many ways can a 2D circle be embedded in a 3D space
- DNA
- Plasma reconnection
- It's fun and makes me feel like a wizard.

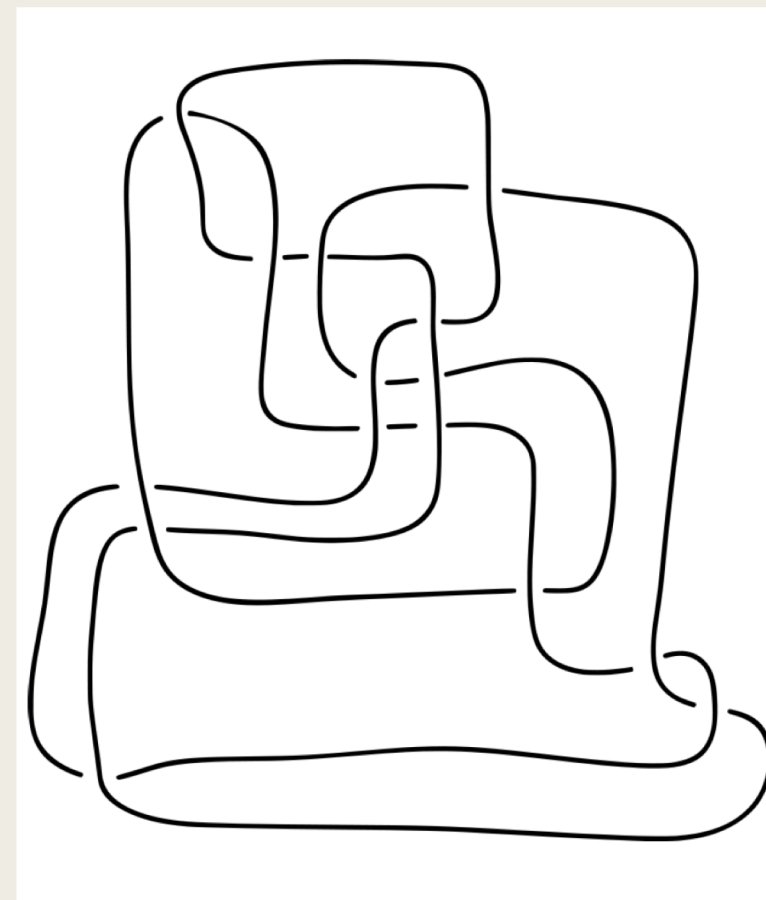
Knot diagrams



Hopf Link

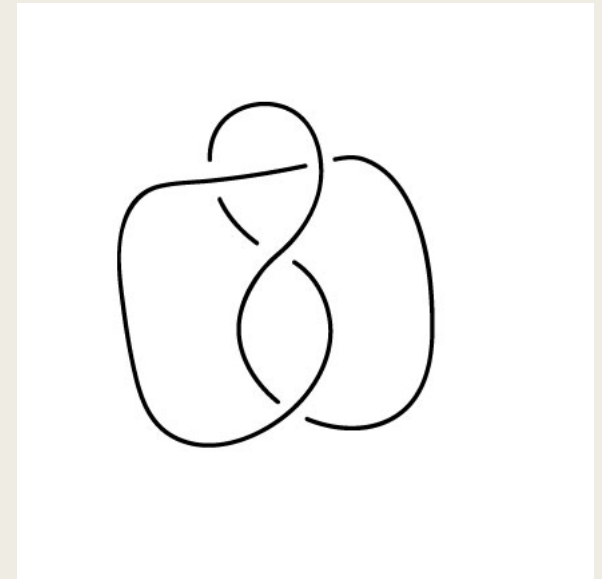
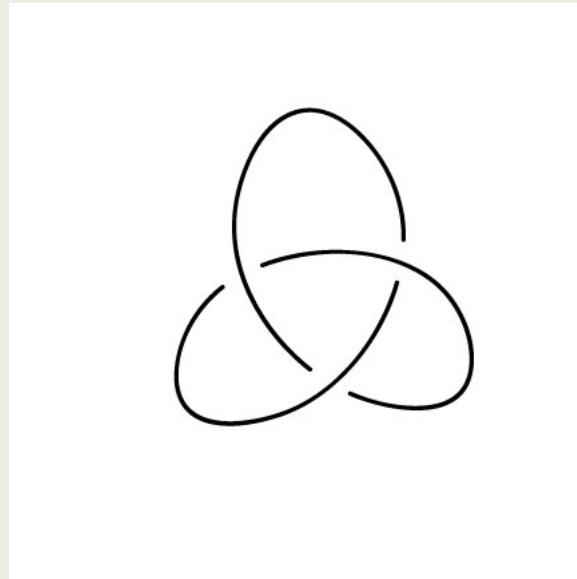
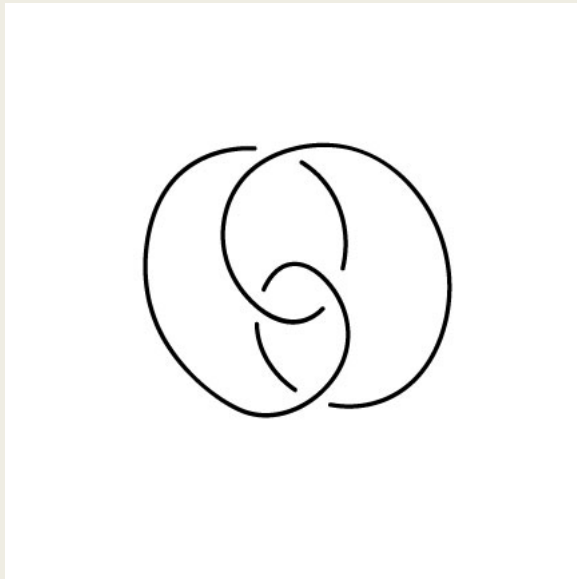
Types of knots:

Unknot



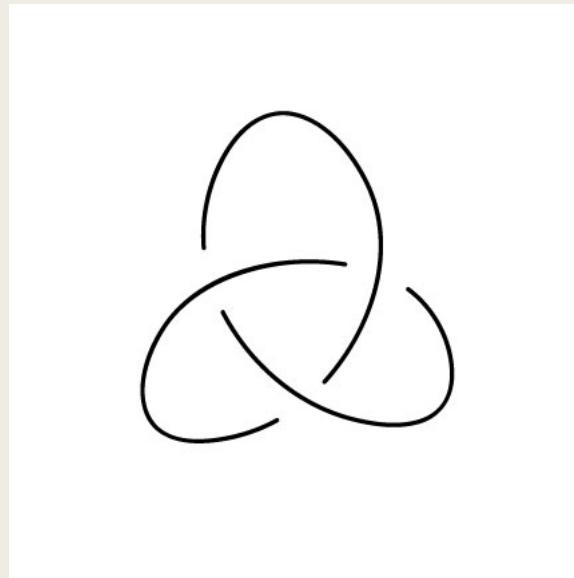
Types of knots:

Left-handed trefoil



Types of knots:

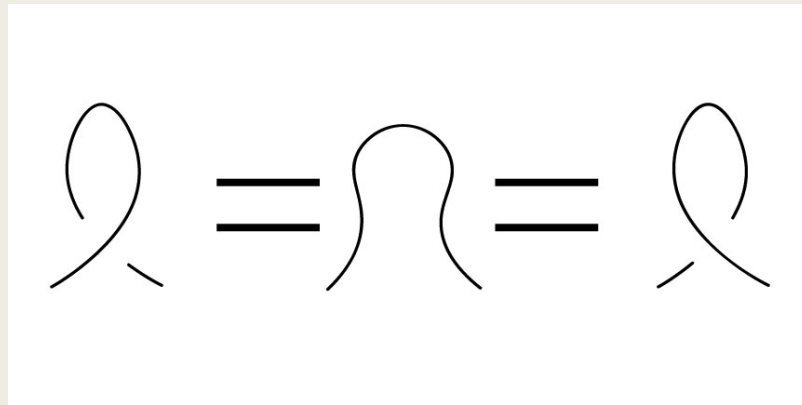
Right-handed trefoil



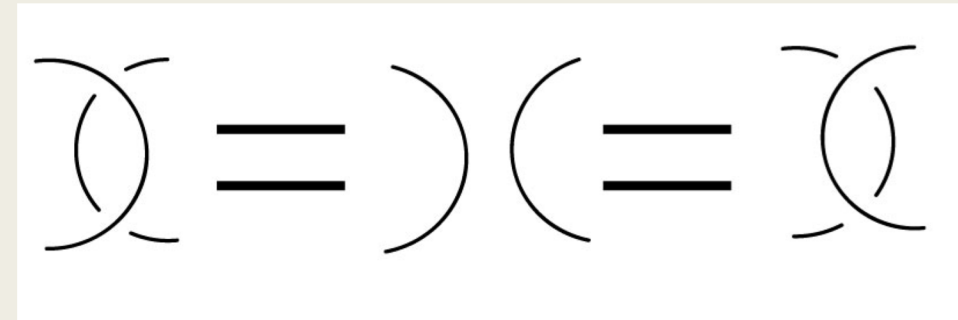
Reidemeister Moves

(equivalence relations)

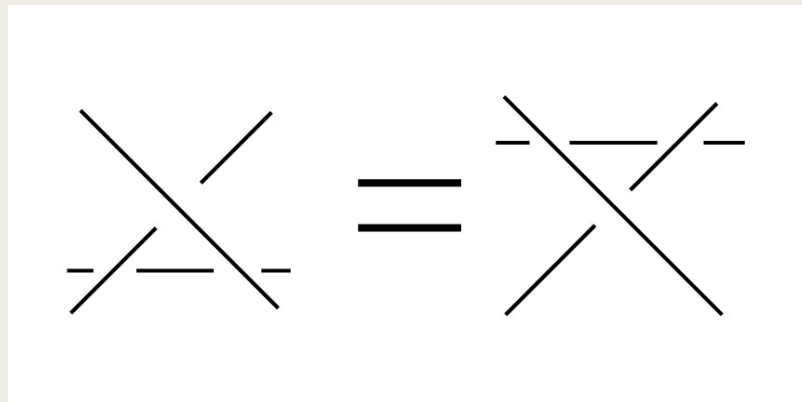
■ R1:



■ R2:



■ R3:



...how is this math?

Polynomials

- Examples of polynomials:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,$$

$$2x + 1, ax^2 + bx + c$$

- Polynomials of a knot, D_1 , is represented by

$$J(D_1)$$

Polynomial Invariants

- Goal: We want the polynomial of two equivalent knots to be invariant under the Reidemeister moves.
- Let D_1 be a knot, let D_2 be a knot after R1, R2, R3. Then,

$$J(D_1) = J(D_2)$$

- We now search for the polynomial rules to suit this sweet property!
 - *Bracket Polynomials*
 - *Jones Polynomials*

Bracket Polynomials

- The rules:

1 :

$$\langle \text{X} \rangle = a \langle \text{) (} \rangle + b \langle \text{) (} \rangle ;$$

2 : $\langle DO \rangle = c \langle D \rangle ;$

3 : $\langle O \rangle = 1.$

Bracket Polynomials

- The rules, now invariant under R2 and R3:

1 :

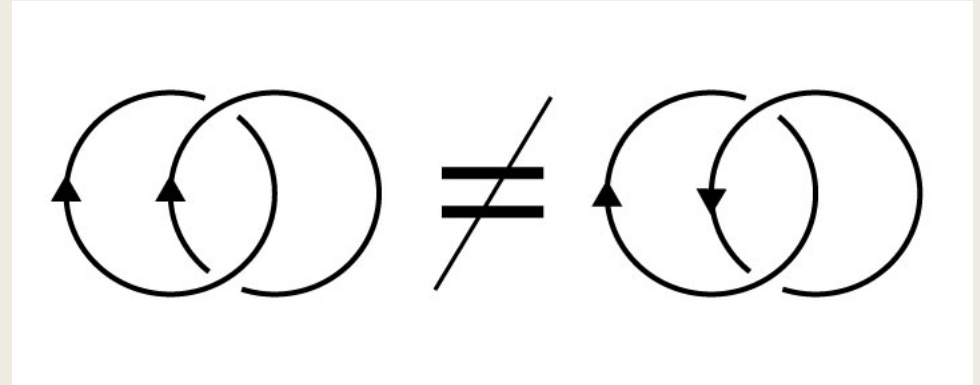
$$\langle \text{crossing} \rangle = a \langle \text{right crossing} \rangle + a^{-1} \langle \text{left crossing} \rangle$$

2 : $\langle DO \rangle = (-a^2 - a^{-2}) \langle D \rangle$

3 : $\langle O \rangle = 1$

...what about R1?

Jones Polynomials



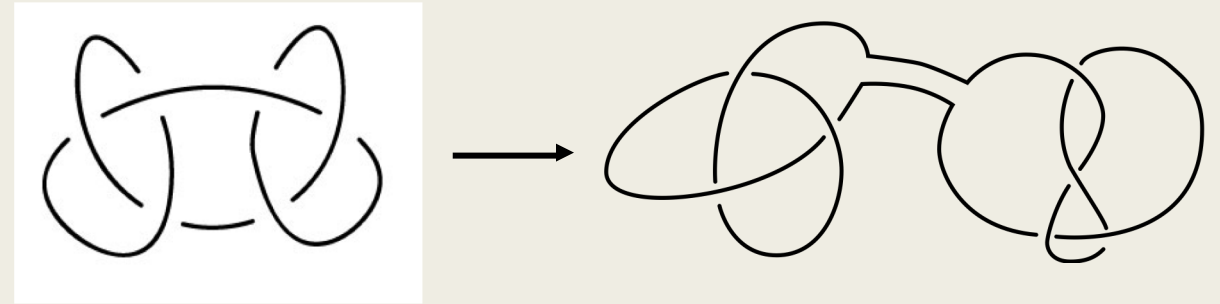
- Apply orientation to our knots.
- Represent knots with the Kauffman Polynomial

$$X(D) = (-a)^{-3w(D)} \langle D \rangle$$

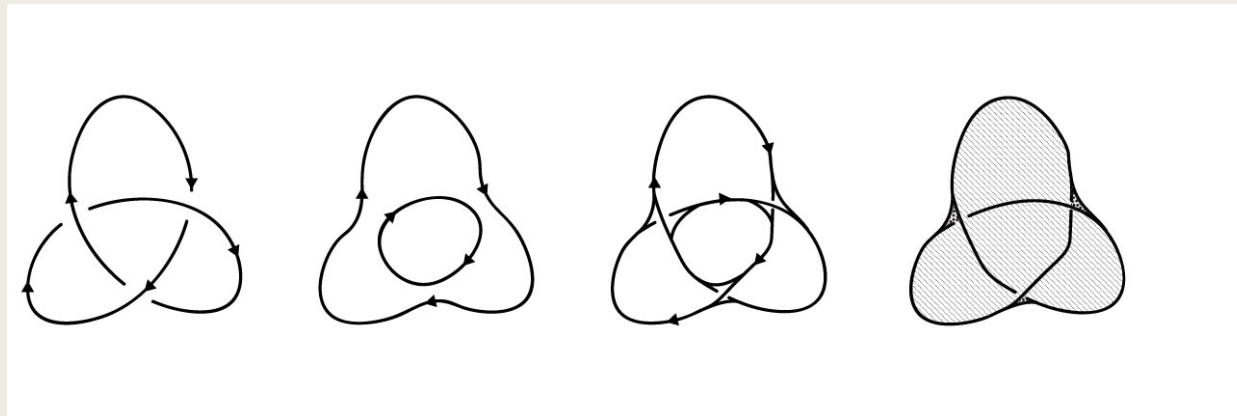
where

$$w(\nearrow) = +1, w(\searrow) = -1$$

Knot Addition

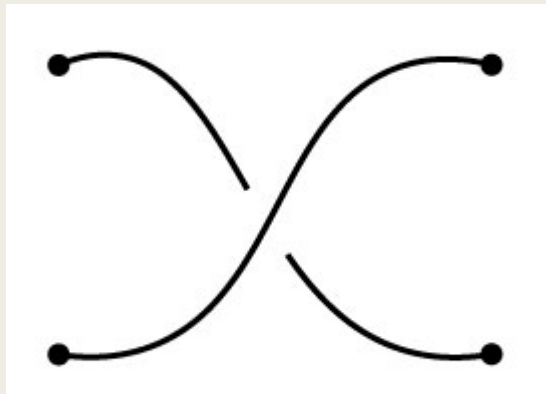


- Knots can be made from adding two other knots!
- Mathematically you add knots by multiplying their polynomials.
- What are prime knots?
 - *Knots like numbers, can be broken into prime factors.*
 - *Examples of prime knots: trefoils, Hopf link*
- Seifert Surfaces

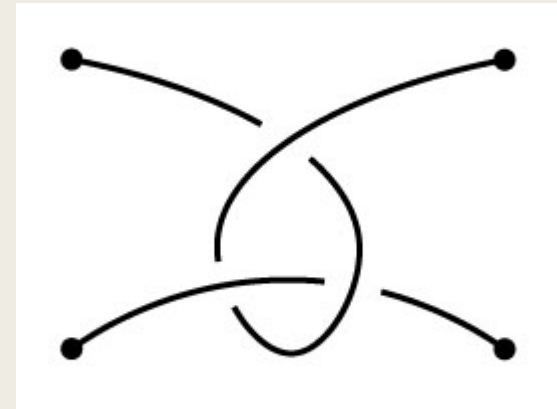


Braids

- Definition: A **braid** of n strands is a collection of $2n$ points, arranged in two columns, connected by n strings. Strings travel from right to left (there is no “double-back”).



Braid



Not a braid

Braids

- How many braids can you have with one strand?
 - *Only one!*
- How many braids can you have on two strands?
 - *Infinitely many!*