‘Epidemiology’ Modeling

Cara Peters

Girls Talk Math
University of Maryland, College Park

July 19, 2018
Outline

What is Mathematical Modeling?

The UMD Bug!

Model of CML with Immune Response
Outline

What is Mathematical Modeling?

The UMD Bug!

Model of CML with Immune Response
What is Mathematical Modeling?

- The process of representing real world problems using mathematical structures
- Why do we use mathematical models?
  - To guide design/control of an experiment
  - To gain understanding of a system
  - To answer questions about the application area
- What can we model?
  - Lots of things!
  - Weather, Cancer Growth/Treatment, Traffic Patterns, Elections, Stock Market, etc.
Building a Mathematical Model

What do we need to consider?

- Goals...what problem are we trying to solve?
- Background information and assumptions
- Components of the model
- Relevant relationships between components
- Relevant quantities and precise values (data)
- Restrictions on relevant quantities
Outline

What is Mathematical Modeling?

The UMD Bug!

Model of CML with Immune Response
Rules of the Game

An infection is spreading around campus....

1. GTM is the origin. Any person that visits the GTM classroom collects two stickers, one red and one blue.
2. This person is now a carrier and chooses to stick one color on their shirt and keep the other color safe for later.
3. Out on campus...if a red carrier meets a blue carrier (or vice versa), they exchange their saved stickers and place the second sticker on their shirts. They are both now infected (wearing two stickers)!
4. Infected persons receive two new stickers (red and blue) and can infect anyone!
   - Give a second sticker to a red or blue carrier.
   - Give both stickers to an unsuspecting person.
Modeling the UMD Epidemic

- What are the components?
  - Susceptible
  - Blue Carrier
  - Red Carrier
  - Infected

- How are they related?
Improvements?

What is missing?
- Infected people can infect Red and Blue Carriers
- Infected people can infect susceptible
- What if someone loses a sticker? Or is "healed"?
- What happens when someone leaves campus?
- What are the interaction rates between the compartments?

How can we include these?
Any other improvements?
Outline

What is Mathematical Modeling?

The UMD Bug!

Model of CML with Immune Response
Chronic Myelogenous Leukemia (CML)

- Cancer of the blood - white blood cells
- Characterized by the Philadelphia chromosome
- Effective treatment exists, but no cure
The Model (Clapp et al. 2015)

A model of CML at the cellular level

How do we translate this into math?
The Model Equations (Clapp et al. 2015)

\[ \dot{y}_0 = b_1 y_1 - a_0 y_0 - \frac{\mu y_0 z}{1 + \varepsilon y_3^2} \]

\[ \dot{y}_1 = a_0 y_0 - b_1 y_1 + r y_1 (1 - \frac{y_1}{K}) - d_1 y_1 - \frac{\mu y_1 z}{1 + \varepsilon y_3^2} \]

\[ \dot{y}_2 = \frac{a_1}{inh_1} y_1 - d_2 y_2 - \frac{\mu y_2 z}{1 + \varepsilon y_3^2} \]

\[ \dot{y}_3 = \frac{a_2}{inh_2} y_2 - d_3 y_3 - \frac{\mu y_3 z}{1 + \varepsilon y_3^2} \]

\[ \dot{z} = s_z - d_z z + \frac{\alpha y_3 z}{1 + \varepsilon y_3^2} \]
The Model Results (Clapp et al. 2015)

What does the model tell us about the BCR-ABL ratio (a ratio of leukemic cells to healthy cells)?
A Simplified Model

- Reduce model to three compartments
- Allows for mathematical analysis
  - Steady states
  - Stability and sensitivity

\[
\begin{align*}
\dot{y}_1 &= ry_1 \left(1 - \frac{y_1}{K}\right) - \mu y_1 z \\
\dot{y}_2 &= a'_1 y_1 - d_2 y_2 - \mu y_2 z \\
\dot{z} &= s - dz + \frac{\alpha y_2 z}{1 + \epsilon y_2^2}
\end{align*}
\]
An Expanded Model

\[ \dot{y}_0 = b_1 y_1 - a_0 y_0 - \frac{\mu y_0 T_1}{1 + \epsilon y_3^2} \]

\[ \dot{y}_1 = a_0 y_0 - b_1 y_1 + r y_1 (1 - \frac{y_1}{K}) - d_1 y_1 - \frac{\mu y_1 T_1}{1 + \epsilon y_3^2} \]

\[ \dot{y}_2 = \frac{a_1}{inh_1} y_1 - d_2 y_2 - \frac{\mu y_2 T_1}{1 + \epsilon y_3^2} \]

\[ \dot{y}_3 = \frac{a_2}{inh_2} y_2 - d_3 y_3 - \frac{\mu y_3 T_1}{1 + \epsilon y_3^2} \]

\[ \dot{T}_0 = s_T - d T_0 - \frac{\alpha y_3 T_0}{1 + \epsilon y_3^2} \]

\[ \dot{T}_1 = \frac{\alpha y_3 T_0}{1 + \epsilon y_3^2} + \alpha y_3 T_1 - (d_{T_1} - r_T) T_1 - k_T R T \]

\[ \dot{R} = r_T T_1 - d_{T_1} R \]
Results of Simulation

- Left: Expanded Model gives almost identical results as the original
- Right: How do parameter values affect the result?
Thank You!

Questions?